The Recoils of a Dynamical Mirror and the Decoherence of its Fluxes

R. Parentani[1]

Laboratoire de Physique Théorique de l'Ecole Normale Supérieure[2]
24 rue Lhomond 75.231 Paris CEDEX 05, France.

Abstract In order to address the problem of the validity of the "background field approximation", we introduce a dynamical model for a mirror described by a massive quantum field. We then analyze the properties of the scattering of a massless field from this dynamical mirror and compare the results with the corresponding quantities evaluated using the original Davies Fulling model in which the mirror is represented by a boundary condition imposed on the massless field at its surface. We show that in certain circumstances, the recoils of the dynamical mirror induce decoherence effects which subsist even when the mass of the mirror is sent to infinity. In particular we study the case of the uniformly accelerated mirror and prove that, after a certain lapse of proper time, the decoherence effects inevitably dominate the physics of the quanta emitted forward. Then, the vanishing of the mean flux obtained in the Davies Fulling model is no longer found but replaced by a positive incoherent flux.

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[1]e-mail: parenta@physique.ens.fr
[2]Unité propre de recherche du C.N.R.S. associée à l' ENS et à l'Université de Paris Sud.

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1 Introduction

In deriving quantum black hole radiance, Hawking[1] realized from the outset that the frequencies defined on $\mathcal{I}^-$ and involved in the Bogoliubov coefficients relating out-modes to in-modes grow exponentially fast with the retarded time around which the out-particle is created. Accordingly Hawking justifies the use of "geometrical optics" to describe propagation through the star. For a recent presentation of this point of view see ref. [2].

More recently there have been debates[3][4][5] on the relevance of these "transplanckian" frequencies when one wishes to take into account gravitational backreaction effects beyond the semi-classical theory wherein only the mean value of the energy momentum tensor acts as a source in Einstein's equations[6][7][8][9]. We remind the reader that the quantum averaged stress tensor is perfectly regular[10] in the evaporating geometry (until the residual black hole mass approaches the Planck mass) and that an infalling observer would detect no quantum characterized by these high frequencies as it crosses the horizon[1].

Whatever is the reader's opinion, the fact is that when ones uses the two assumptions made by Hawking, to wit *free* propagation in a *given* geometry, one finds quantum correlations between the detection of an asymptotic quantum and the configurations at early times which are characterized by these frequencies. These correlations show up in various ways. As said above, they appear in the Bogoliubov coefficients. It is nevertheless not straightforward to understand the meaning nor the consequences of this presence. More explicit is the calculation of the commutators between local operators and asymptotic annihilation operator as proposed in ref. [11] or equivalently the calculation of off-diagonal matrix elements of $T_{\mu\nu}$ as done in refs. [12][13][14]. Indeed, in a perturbative treatment of the gravitational back-reaction, these matrix elements do intervene in the first order corrections.

The challenging problem is therefore to determine when the gravitational effects induced by these fluctuations and absent in the semi-classical treatment will invalidate the predictions of the latter. The problem is not to ask *in abstracto* whether the background field approximation is valid or not. Rather, one should work with different schemes of calculation to approach the fully quantized theory. Then one tests the validity of the semi-classical treatment by determining the duration of time for which it furnishes a correct estimate of the matrix elements of the relevant observables.

The goal of this paper is to answer this type of questions in the simpler context of the radiation spontaneously emitted by a non inertial mirror. Two different schemes will be compared. First, we shall reexamine the original Davies and Fulling (DF) model[15] in which the trajectory of the mirror is classical and specified once for all. This corresponds to the background field approximation without backreaction. Secondly, we shall introduce a quantized version in which the mirror is described by a massive scalar quantum field; see ref. [16] for a similar quantization in the case of the accelerated detector. In this second scheme, the mirror recoils according to Feyman rules when it scatters a quantum or when it creates a pair of quanta. In this we differ from Chung and Verlinde[17] who work with a hybrid (semi-classical) model in which the source of the recoil is the mean energy momentum tensor expressed in terms of the (first) quantized

mirror's trajectory. This point is further taken up in Section 5. We then determine from our formalism what is the subset of physical quantities correctly estimated in the DF model in various circumstances. It will be seen that the matrix elements which are sensitive to interferences (i.e. sensitive to relative phases between states characterized by different particle number) will be the first affected by the recoils of the heavy mirror. On the contrary, those expressed as sum of squares will be almost unaffected by the recoils and therefore almost identical to the ones obtained in the DF model.

We have organized the paper as follows. In Section 2 we review the properties of the DF model emphasizing the fluctuating properties of the flux spontaneously emitted.

In Section 3, we analyze the flux emitted by a uniformly accelerated mirror. It was shown by Davies and Fulling that the mean flux vanishes even though there are emissions of quanta, see [18][19]. We show in detail how these antagonistic properties can be understood by introducing the concept of a partially reflecting mirror which allows us to decouple adiabatically the field from the mirror for asymptotic times. Furthermore, we relate these properties to Unruh effect[20] and to the radiation emitted by a thermalized accelerated atom which enjoys similar properties[21][22][23][24][13].

In Section 4, we introduce our model for an inertial dynamical mirror based on quantum field theory. Since the mirror is inertial there is no production but the scattering is however not trivial. We show how this dynamical model is related to the DF model by taking the large mass limit of the S matrix elements and then resumming the Born series[25] so as to recover the overlap between in and out states evaluated in the DF model. Nevertheless, in spite of this strict correspondence, interferences effects between the radiation and the mirror wave functions are induced by recoil (i.e. the momentum transfer upon reflection) and modify the expectation values of local operators when the radiation is characterized by a fluctuating particle number. Would the radiation be described by a diagonal matrix density or by a pure state with a definite particle number, no such effect would be found in the large mass limit.

In Section 5, we generalize the dynamical model to non-inertial trajectories by introducing an external electric field which brings a charged mirror into non-inertial motion. We reexamine the case of constant acceleration and compare the results with the ones obtained in Section 3. We see how the recoils induced by the production acts modify the properties of the flux. The main consequence is that the vanishing of the mean flux can no longer be maintained for an arbitrarily long period. After a proper time lapse given by $(\ln M/a)/2a$, where a is the acceleration and M the rest mass of the mirror, the quanta emitted forward decohere and the flux of energy is positive. Furthermore, the notion of the "partner" of a specific quantum (particles are always created in pairs) is also affected by the dynamical character of the mirror. Indeed, the mirror gets correlated to the constituents of each pair by the momentum transfer at the creation act. To describe these correlations we consider the conditional value[13] of the energy momentum tensor correlated to the detection of a specific quantum. This quantity which is expressed as an off-diagonal matrix element of $T_{\mu\nu}$ is much more sensitive to the dynamics of the mirror than the mean value. It is therefore a useful tool to investigate the role of the dynamics of the mirror and as a byproduct to understand the nature and the validity of the "background field approximation" represented here by the DF model.

2   The Scattering Amplitudes and the Fluxes in the Absence of Recoil

In this section, we review the properties of the Davies-Fulling model[15] wherein a massless quantum field is scattered by a moving mirror which follows a given classical trajectory. We emphasize the relations between the classical and the quantum aspects of this scattering because they illustrate the nature of the approximations implicitly used in the DF model when one considers it as the large mass limit of some dynamical model. For the same reason, we also describe the fluctuating properties of the flux spontaneously emitted by the mirror. The reader may consult refs. [15][18][19] for discussions of others aspects of the DF model.

Following Davies and Fulling [15], we consider the quantum mechanical problem defined by the two equations

$$\left[\partial_t^2 - \partial_z^2\right]\phi(t,z) = 0 \tag{1}$$

and

$$\phi(t, z_{cl}(t)) = 0 \tag{2}$$

where ϕ is a complex field and $z_{cl}(t)$ is the trajectory of the "mirror" expressed in carthesian coordinates t and z. Since we work in $1+1$ dimensions, it is appropriate to introduce the light-like coordinates U and V defined by $U, V = t \mp z$. In these coordinates eq. (1) becomes $\partial_U \partial_V \phi(U, V) = 0$, hence the general solution is the sum of functions of U or V only.

Owing to the linearity of the both eqs. (1) and (2) *and* the classical character of the given trajectory, the quantum scattering amplitudes can be obtained in purely classical terms in the sense that $\hbar$ does not appear. We shall therefore first analyze the classical theory of the scattering of ϕ and then see how the classical concepts are reinterpreted in second quantization. This exercise will be found useful when, in the next sections, we shall modify eqs. (1) and (2) to give a dynamical content to the mirror's motion. In particular it will allow us to identify the corrections which are quantum.

In this section, for simplicity, the mirror's trajectory is taken to be inertial and time-like at $t = \pm\infty$. In conformal terms, it means that it starts in i^- and ends up in i^+[26], the time-like past and future infinities. In scattering terms, it means that there is a complete decoupling of what happens to the right and to the left. Therefore the current of an ingoing flux is fully recovered in the scattered outgoing flux and *vice versa* if one studies propagation backwards in time. When the trajectory does not end up at i^+, there is, so to speak, the formation of a horizon and a part only of the incident flux is reflected. This situation will be discussed in the next section.

We first study the scattering to the right of a left moving in-mode of frequency ω

$$\varphi_\omega(V) = \frac{e^{-i\omega V}}{\sqrt{4\pi|\omega|}} \tag{3}$$

This mode carries a conserved current $J_V = \varphi_\omega^* i \overleftrightarrow{\partial_V} \varphi_\omega = 1/2\pi$ and its norm is given by

$$< \varphi_{\omega'},\ \varphi_\omega > = \int_{-\infty}^{\infty} dV \varphi_{\omega'}^* i \overleftrightarrow{\partial_V} \varphi_\omega = \text{sign}(\omega)\delta(\omega' - \omega) \tag{4}$$

Upon reflection on the mirror whose trajectory is now expressed as $V = V_{cl}(U)$, the scattered mode is

$$\varphi_\omega^{scat}(U) = -\frac{e^{-i\omega V_{cl}(U)}}{\sqrt{4\pi|\omega|}} \tag{5}$$

On $\mathcal{I}^+$, i.e. on $V = \infty$, one can decompose this scattered mode in Fourrier transform in terms of the right moving out-modes

$$\varphi_\lambda(U) = \frac{e^{-i\lambda U}}{\sqrt{4\pi|\lambda|}} \tag{6}$$

which form a complete set if λ span the whole real axe:

$$\varphi_\omega^{scat}(U) = \int_{-\infty}^{\infty} d\lambda \gamma_{\omega,\lambda} \varphi_\lambda(U) \tag{7}$$

where the coefficents $\gamma_{\omega,\lambda}$ are given by

$$\begin{aligned} \gamma_{\omega,\lambda} &= \text{sign}(\lambda) < \varphi_\lambda, \ \varphi_\omega > \\ &= -2 \int_{-\infty}^{\infty} dU \frac{e^{i\lambda U}}{\sqrt{4\pi|\lambda|^{-1}}} \frac{e^{-i\omega V_{cl}(U)}}{\sqrt{4\pi|\omega|}} \end{aligned} \tag{8}$$

We have used eq. (4) and integrated by parts. Eqs. (7) and (8) express the conventional Fourrier decomposition with the relativistic weight $1/\sqrt{4\pi|\lambda|}$ included. This writting together with the conservation of the current on the mirror implies that the coefficients $\gamma_{\omega,\lambda}$ satisfy the following "unitary" relations

$$\begin{aligned} \int_0^{\infty} d\lambda \left[\gamma_{\omega,\lambda} \gamma_{\omega',\lambda}^* - \gamma_{\omega,-\lambda} \gamma_{\omega',-\lambda}^* \right] &= \delta(\omega' - \omega) \\ \int_0^{\infty} d\omega \left[\gamma_{\omega,\lambda'} \gamma_{\omega,\lambda}^* - \gamma_{-\omega,\lambda'}^* \gamma_{-\omega,\lambda} \right] &= \delta(\lambda' - \lambda) \end{aligned} \tag{9}$$

In classical mechanics, the energy density of an incident wave of frequency ω and amplitude A_ω is

$$T_{VV}^{cl} = 2|A_\omega|^2 \ |\partial_V \varphi_\omega(V)|^2 = |A_\omega|^2 \frac{|\omega|}{2\pi} \tag{10}$$

Using eq. (7), the energy density of the scattered mode reads

$$\begin{aligned} T_{UU}^{cl} &= 2|A_\omega|^2 \ |\partial_U \varphi_\omega^{scat}(U)|^2 \\ &= |A_\omega|^2 \int_0^{\infty} d\lambda \int_0^{\infty} d\lambda' \frac{\sqrt{\lambda\lambda'}}{2\pi} \\ &\quad \left[e^{-i(\lambda-\lambda')U} \left(\gamma_{\omega,\lambda} \gamma_{\omega,\lambda'}^* + \gamma_{\omega,-\lambda}^* \gamma_{\omega,-\lambda'} \right) - 2\text{Re} \left(e^{-i(\lambda+\lambda')U} \gamma_{\omega,\lambda} \gamma_{\omega,-\lambda'}^* \right) \right] \end{aligned} \tag{11}$$

Notice that the second term does not contribute to the total scattered energy defined by $H_U = \int_{-\infty}^{\infty} dU T_{UU}^{cl}$.

In order to get an approximate temporal description, we introduce broad wave packets defined by

$$\varphi_{\bar{\omega}}(V) = \int_{-\infty}^{\infty} d\omega' f_{\bar{\omega}}(\omega') \varphi_{\omega'}(V) \tag{12}$$

These waves carry unit charge if $\int d\omega |f_{\bar{\omega}}(\omega')|^2 = 1$. They are broad if the spread around the mean frequency $\bar{\omega}$ is much smaller than $\bar{\omega}$ itself. Broad wave packets of the form $A_{\bar{\omega}} \varphi_{\bar{\omega}}$ carry an energy given by $H_V = \int dV T_{VV}^{cl} = |A_{\bar{\omega}}|^2 |\bar{\omega}|$.

The classical relation between the Doppler shifted frequency of the scattered mode and the out-frequency λ is recovered in this formalism upon computing the stationary phase in the integrant of eq. (7). One obtains

$$\omega \frac{dV_{cl}(U)}{dU} = \lambda \tag{13}$$

At fixed λ and ω, it indicates at which value of U does ω resonates with λ. (Remember that $dV_{cl}/dU > 0$ holds for time like trajectories.) Therefore, when the rate of change of the trajectory is small on the scale of the inverse frequency of the incoming wave packet, i.e. when the adiabatic (W.K.B.) approximation is valid, the mean frequency of the scattered mode issuing from a broad wave packet centered along $V = V_0$ and of mean frequency $\bar{\omega}$ is given by $\bar{\lambda} = \bar{\omega} dV_{cl}/dU$ evaluated on $V = V_0$. The mirror acts therefore as an infinite reservoir of energy and momentum which leads to these relations no matter what A_{ω} and ω are. Upon introducing some dynamics, we shall see that the rest mass of the mirror will modify these relations for incoming waves with large energy, i.e. comparable to the mirror's mass.

Before studying the quantum scattering, we notice that, for broad wave packets, the charge carried by the first term of eq. (9) is greather than the incident one. A similar increase of the reflected wave found in a Kerr geometry for certain angular momenta[27] was called super-radiance by analogy with induced emission.

Quantum Scattering

In second quantization, ϕ becomes a field operator. In the Heisenberg representation, it is decomposed in positive and negative frequency parts according to

$$\phi(U,V) = \int_0^{\infty} d\omega \left(a_{\omega} \left[\varphi_{\omega}(V) + \varphi_{\omega}^{scat}(U) \right] + b_{\omega}^{\dagger} \left[\varphi_{\omega}^*(V) + \varphi_{\omega}^{*\,scat}(U) \right] \right) \tag{14}$$

The annihilation operators a_{ω}, b_{ω} define the in-vacuum by $a_{\omega}|0,in\rangle = b_{\omega}|0,in\rangle = 0$ [15][18]. This in vacuum corresponds to the usual Minkowski vacuum on $\mathcal{I}^-$ since the trajectory of the mirror is asymptotically inertial.

Similarly one decomposes ϕ in the out-basis

$$\phi(U,V) = \int_0^{\infty} d\lambda \left(a_{\lambda} \left[\varphi_{\lambda}(U) + \varphi_{\lambda}^{bscat}(V) \right] + b_{\lambda}^{\dagger} \left[\varphi_{\lambda}^*(U) + \varphi_{\lambda}^{*\,bscat}(V) \right] \right) \tag{15}$$

where the a_{λ} and the b_{λ} operators define the out-vacuum and where $\varphi_{\lambda}^{bscat}(V)$ is the back scattered wave obtained by propagating backwards the out-mode $\varphi_{\lambda}(U)$ and imposing

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the reflection condition, eq. (2). It is given by

$$\varphi_\lambda^{bscat}(V) = -\frac{e^{-i\lambda U_{cl}(V)}}{\sqrt{4\pi|\lambda|}} \tag{16}$$

The a_λ and the b_λ operators define the out-vacuum which correspond to Minkowski vacuum on $\mathcal{I}^+$. The Bogoliubov transformation relates the in operators to the out ones as

$$
\begin{aligned}
a_\omega &= \alpha_{\omega,\lambda}^* a_\lambda + \beta_{\omega,\lambda}^* b_\lambda^\dagger \\
b_\omega &= \alpha_{\omega,\lambda}^* b_\lambda + \beta_{\omega,\lambda}^* a_\lambda^\dagger
\end{aligned} \tag{17}
$$

where the Bogoliubov coefficients $\alpha_{\omega,\lambda}$ and $\beta_{\omega,\lambda}$ are related to the $\gamma_{\omega,\lambda}$ by

$$
\begin{aligned}
\alpha_{\omega,\lambda} &= \gamma_{\omega,\lambda} \\
\beta_{\omega,\lambda} &= \gamma_{\omega,-\lambda}
\end{aligned} \tag{18}
$$

Therefore the values of α and β have nothing to do with quantum mechanics, in particular $\beta_{\omega,\lambda}$ where ω and λ are frequencies and not energies does not become comparable to $\alpha_{\omega,\lambda}$ as $\hbar \to 0$.

In quantum mechanics, the non vanishing character of the β coefficients encodes spontaneous pair creation. Indeed, starting with no particle on $\mathcal{I}^-$, the probability to find no particle on $\mathcal{I}^+$ is no longer unity (as it would have been if the trajectory was inertial) but given by

$$|\langle 0, out|0, in\rangle|^2 = e^{-\Sigma_{\bar\lambda} \ln(1+\langle n\rangle_{\bar\lambda})} = \Pi_{\bar\lambda} \frac{1}{\int d\omega |\alpha_{\omega,\bar\lambda}|^2} = \frac{1}{Z} < 1 \tag{19}$$

where $\langle n\rangle_{\bar\lambda}$ is the mean number of created particle of frequency $\bar\lambda$. In terms of the Bogoliubov coefficients it is given by

$$\langle n_{\bar\lambda}\rangle = \langle 0, in|a_{\bar\lambda}^\dagger a_{\bar\lambda}|0, in\rangle = \int_0^\infty d\omega |\beta_{\omega,\bar\lambda}|^2 \ (= \langle 0, in|b_{\bar\lambda}^\dagger b_{\bar\lambda}|0, in\rangle) \tag{20}$$

We have used for conveniance of complete set of broad wave packets labelled by $\bar\lambda$ to represent the out states. Their usual normalization in terms of δ of Kronecker simplify both the equations and their interpreation.

Returning to plane waves, one defines the mean density by $\langle n_\lambda\rangle d\lambda = \langle n_{\bar\lambda}\rangle$. Then, in terms of this density, the mean energy emitted by the mirror is given by

$$\langle H_U^{scat}\rangle = 2 \int_0^\infty d\lambda \lambda \langle n_\lambda\rangle = \int_0^\infty d\lambda \lambda \langle 0, in| \left(a_\lambda^\dagger a_\lambda + b_\lambda^\dagger b_\lambda\right) |0, in\rangle = \int_{-\infty}^\infty dU \langle T_{UU}^{scat}\rangle \tag{21}$$

where $\langle T_{UU}^{scat}\rangle$ is the mean flux given by

$$
\begin{aligned}
\langle T_{UU}^{scat}\rangle &= \langle 0, in| \left(\partial_U \phi^\dagger \partial_U \phi + \partial_U \phi \partial_U \phi^\dagger\right) |0, in\rangle - \langle 0, out| \left(\partial_U \phi^\dagger \partial_U \phi + \partial_U \phi \partial_U \phi^\dagger\right) |0, out\rangle \\
&= 2 \int_0^\infty d\lambda \int_0^\infty d\lambda' \frac{\sqrt{\lambda\lambda'}}{2\pi} \\
&\quad \left[e^{-i(\lambda-\lambda')U} \left(\int_0^\infty d\omega \beta_{\omega,\lambda}^* \beta_{\omega,\lambda'}\right) - \mathrm{Re}\left(e^{-i(\lambda+\lambda')U} \int_0^\infty d\omega \alpha_{\omega,\lambda} \beta_{\omega,\lambda'}^*\right)\right] \tag{22}
\end{aligned}
$$

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where the Minkowski zero point energy has been subtracted. Upon comparaison with the classical flux carried by the scattered mode φ_ω^{scat}, eq. (11), one sees that in quantum mechanics, the mean flux, i.e. the quantum averaged flux, is given by the sum of the classical excesses of the each incoming mode of amplitude squared $|A_\omega|^2 = \hbar = 1$. Indeed the first term of eq. (11) when summed over all positive ω gives the zero point energy plus the first term of eq. (22).

Furthermore, as pointed out in ref. [15], this mean flux can be expressed in terms of the local properties of the trajectory $V = V_{cl}(U)$ only. Using Green (Wightman) functions given by

$$
\begin{aligned}
\langle 0, in|\phi^\dagger(x)\phi(x')|0, in\rangle &= \int_0^\infty \frac{d\omega}{4\pi\omega} \left[e^{i\omega(V'-V)} + e^{i\omega[V_{cl}(U')-V_{cl}(U)]} \right] \\
\langle 0, out|\phi^\dagger(x)\phi(x')|0, out\rangle &= \int_0^\infty \frac{d\lambda}{4\pi\lambda} \left[e^{i\lambda[U_{cl}(V')-U_{cl}(V)]} + e^{i\lambda(U'-U)} \right]
\end{aligned}
\tag{23}
$$

one obtains

$$
\begin{aligned}
\langle T_{UU}^{scat} \rangle &= 2 \lim_{U' \to U} \left[\langle 0, in|\partial_{U'}\phi^\dagger \partial_U \phi|0, in\rangle - \langle 0, out|\partial_{U'}\phi^\dagger \partial_U \phi|0, out\rangle \right] \\
&= 2 \lim_{U' \to U} -\frac{1}{4\pi} \partial_{U'}\partial_U \left[\ln|V_{cl}(U') - V_{cl}(U)| - \ln|U' - U| \right] \\
&= -\frac{1}{6\pi} \left[\left(\frac{dV_{cl}}{dU} \right)^{1/2} \partial_U^2 \left(\frac{dV_{cl}}{dU} \right)^{-1/2} \right]
\end{aligned}
\tag{24}
$$

Very important is the fact that this equation can be rewritten as

$$
\langle T_{UU}^{scat} \rangle = \frac{1}{12\pi} \left[\left(\frac{d^2 V_{cl}}{dU^2} \right) \left(\frac{dV_{cl}}{dU} \right)^{-1} \right]^2 - \frac{1}{6\pi} \partial_U \left[\left(\frac{d^2 V_{cl}}{dU^2} \right) \left(\frac{dV_{cl}}{dU} \right)^{-1} \right]
\tag{25}
$$

Only the first term, positive definite, contributes to $\langle H \rangle$ of eq. (21). Indeed for asymptotically inertial trajectories, the second term integrates to zero. Similarly, the second term of the decomposition of the r.h.s. in eq. (22) intregrates also to zero. This latter comes from interferences betwen final states with differents particle numbers. We recall that T_{UU} contains fluctuating terms made of products of ab or $a^\dagger b^\dagger$. These products of operators give rise to the second term of eq. (22). One sees therefore that the *mean* flux encodes contributions from interferences which find their origin in the rewriting of the in-vacuum in terms of out-states. Notice the similarity between the decomposition of eq. (22) and the one of eq. (25). In both cases, the first term is positive definite and the second one gives no contribution to the total energy.

We now mention some properties of the fluctuations of the out-particle content of the in-vacuum. There are many ways to describe these fluctuations. For instance, one can pick a particular final state $|\psi\rangle$ and inquire into the probability to find it on $\mathcal{I}^+$. This probability is given by

$$
P_\psi = \langle 0, in|\psi\rangle\langle\psi|0, in\rangle
\tag{26}
$$

Notice that the specification of the final state needs not to be complete. Indeed one can partially fix the final state by using projectors on subspaces of the Fock space. In order

to display the correlations among constituents of each pair of quanta, we now consider these partial specifications. The reason for this kinematical exercise is to compare the correlations in this model with the ones that we shall find in the dynamical model. We shall see that the notion of "partner" differs significantly since, in the dynamical version, the field configurations are correlated to the mirror's state as well.

We choose to specify only the particle content of the final state and we trace over the states of anti-particle sector. The selected particle is described by the state

$$|\bar{\lambda}\rangle = \int_0^\infty d\lambda \; f_\lambda a_\lambda^\dagger |0_a, out\rangle \tag{27}$$

where $|0_a, out\rangle$ is the particle sector of the out vacuum on which the a operators act. The projector which specifies that there is this particle and which does not specify the state of the antiparticle sector is[13]

$$\Pi_{\bar{\lambda}} = \; \mathrm{I}_b \otimes \int_0^\infty d\lambda \; f_\lambda a_\lambda^\dagger |0_a, out\rangle\langle 0_a, out| \int_0^\infty d\lambda' \; f_{\lambda'}^* a_{\lambda'} \tag{28}$$

where I_b is the operator unity acting on the anti-particle b-sector of the final state. A simple algebra gives the probability to find this state

$$P_{\bar{\lambda}} = \langle 0, in|\Pi_{\bar{\lambda}}|0, in\rangle = \frac{1}{Z} \int_0^\infty d\lambda|f_\lambda|^2 \; |\int_0^\infty d\omega \beta_{\omega,\lambda}\alpha_{\omega,\lambda}^{-1}|^2 \tag{29}$$

where $\alpha_{\omega\lambda}^{-1}$ is the inverse matrix of $\alpha_{\omega\lambda}$. The state of the anti-particle correlated to the state $|\bar{\lambda}\rangle$ is defined by $|\bar{\lambda}\rangle|partner, \; \bar{\lambda}\rangle = \Pi_{\bar{\lambda}}|0, in\rangle$ and given by

$$|partner, \; \bar{\lambda}\rangle = \int_0^\infty d\lambda f_\lambda^* \int_0^\infty d\omega \beta_{\omega,\lambda}^*(\alpha_{\omega,\lambda}^{-1})^* \; b_\lambda^\dagger |0_b, out\rangle \tag{30}$$

Owing to the correlations among field configurations in the in-vacuum, the "partner" is therefore uniquely determined by the f_λ's but its wave function is not the complex conjugate of the particle one.

If one selects a two particle state made of orthogonal wave packets and if one still traces over the antiparticle sector, the probability to find this state is the product of the probabilities to find each particle separately. One has thus production of statistically independent pairs of Minkowski quanta whose constituents are perfectly correlated in phase and amplitude.

Since many final outcomes are possible, the flux of energy fluctuates as well since it fluctuates accordingly. To describe these correlated fluctuations, it is appropriate to evaluate the following *off diagonal* matrix elements of T_{UU}[13][28]

$$\langle T_{UU}\rangle_\Pi = \frac{\langle 0, in|\Pi \; T_{UU}|0, in\rangle}{\langle 0, in|\Pi|0, in\rangle} \tag{31}$$

associated to each final outcome specified by the projector Π. This matrix element can be obtained by decomposing the mean flux as

$$\begin{aligned} \langle 0, in|T_{UU}|0, in\rangle &= \langle 0, in|\sum_i \Pi_i T_{UU}|0, in\rangle \\ &= \sum_i P_i \frac{\langle 0, in|\Pi_i T_{UU}|0, in\rangle}{\langle 0, in|\Pi_i|0, in\rangle} \end{aligned} \tag{32}$$

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where the projectors Π_i form a complete set which decomposes the unit operator I as $\sum_i \Pi_i =$I. Eq. (32) decomposes the mean flux according to the final state as the sum of the products of the probability of finding a particular out come times the matrix element associated with this outcome. Each matrix element has therefore the interpretation of the conditional value of the flux knowing that the initial state is $|0, in\rangle$ and the final one $\Pi_i|0, in\rangle$; see refs. [13][29][28] for more details. Let us mention only that owing to the perfect reflection on $V_{cl}(U)$, one can also calculate the conditional value of T_{VV} before scattering by the mirror: $\langle T_{VV}\rangle_\Pi = \langle T_{UU}\rangle_\Pi (dV_{cl}/dU)^{-2}$. Thus $\langle T_{\mu\nu}\rangle_\Pi$ extends from $\mathcal{I}^+$ back to $\mathcal{I}^-$ no matter what are the frequencies involved. Notice that these properties also hold in the black hole case precisely within the context of Hawking's hypothesis of free propagation in a fixed background[12][14]. In the next sections, we shall consider similar off diagonal matrix elements of $T_{\mu\nu}$ in order to isolate the modifications of these correlations induced by the recoils of the dynamical mirror. We shall prove that the correlations to the past are inevitably washed out once the characteristic energy of the fluctuations involved in the matrix element approaches a certain function of the rest mass of the mirror thereby restricting the validity of the results derived using the background field approximation.

3   The Uniformly Accelerating Mirror

In order to illustrate the relations between the fluctuating properties of the radiation, the particle concept and the mean flux, we analyze in detail the flux emitted by a mirror in constant acceleration. This case is simple enough to obtain analytical results but nevertheless possesses intriguing finely tuned interferences which, for instance, lead to the vanishing of the mean flux. We shall generalize the scattering to partially transmitting mirrors and show that the radiation emitted by these mirrors is closely related to the radiation emitted by an accelerated oscillator heated by Unruh effect[20]. In Section 5, we shall reexamine the accelerated mirror with our dynamical model and determine the nature of the modifications induced by the mirror's dynamics. These put severe restrictions on the validity of the DF model.

Constant acceleration a means that the trajectory of the mirror satisfies $V_{cl}(U) = -1/a^2U$. We put the mirror on the left quadrant: $V < 0$, $U > 0$. The trajectory of the mirror does not star at i^- nor end at i^+. Indeed, for $t \to -\infty$, $U \to 0^+$ and for $t \to \infty$, $V \to 0^-$. This fact justifies or even implies, as we shall see, consideration of scattering from both sides from an asymptotically "transparent mirror".

To explore the properties of the scattering, we first consider the mean flux emitted to the right, see eq. (24). Since $(dV_{cl}/dU)^{-1/2} = aU$, $\langle T_{UU}^{scat} \rangle$ vanishes[18][19]. Nevertheless the field configurations are scattered, hence some Minkowski quanta are produced. This can be seen from the non-vanishing character of the β coefficients of the Bogoliubov transformation

$$\alpha_{\omega,\lambda} = -\int_0^\infty \frac{dU}{2\pi} \frac{e^{i\lambda U} e^{i\omega/a^2 U}}{\sqrt{\omega/\lambda}} = \frac{-1}{a\pi} K_1 \left[2i(\frac{\omega\lambda}{a^2})^{1/2} \right]$$

$$\beta_{\omega,\lambda} = -\int_0^\infty \frac{dU}{2\pi} \frac{e^{-i\lambda U} e^{i\omega/a^2 U}}{\sqrt{\omega/\lambda}} = \frac{i}{a\pi} K_1 \left[2(\frac{\omega\lambda}{a^2})^{1/2} \right] \tag{33}$$

where $K_1[z]$ is a modified Bessel function, see ref. [18]. One then obtains that the total energy, eq. (21), associated with those creations diverges. To have simultaneously a vanishing local flux and a diverging total energy whose origin comes from singular transcients is a common feature of the radiation emitted by accelerated systems when the recoils are completely neglected: see [30] for the radiation emitted by a classical charge and [22][23][24][13] for the emission associated with the Unruh effect. In each case, one can decompose the mean flux in two terms as in eq. (25). One finds that the first term describes a positive constant flux in the local frame (in our case one has: $\langle T_{uu} \rangle^{first\ term} = a^2/6\pi$ where $au = \ln aU$ is the local coordinate at rest, see below eq. (35)) and that the second (negative) term, properly regulated, integrates to zero. Hence the transcients have a global character. Indeed they encode the total number of particle created which is a linear function of the lapse of proper time during which the acceleration is constant[13].

Nevertheless, it is not clear how to handle correctly the fact that the in-modes and the scattered modes are discontinuous on $U = 0$. Therefore it is appropriate to gain some insight by computing the two point Green function on $\mathcal{I}^+$. Since the mirror reaches $V = 0$ at $t = \infty$, on the right of it, $\mathcal{I}^+$ is now the union of $U = \infty$, $V > 0$ and $V = \infty$ all U's. Near $\mathcal{I}^+$, the Green function contains three types of terms

$$
\begin{aligned}
\frac{\langle 0, in|\phi^\dagger(x')\phi(x)|0, in\rangle}{-4\pi} = \ & \theta(-U')\theta(-U)\ln|U'-U| + \theta(V')\theta(V)\ln|V'-V| \\
& +\theta(U')\theta(U)\ln|V_{cl}(U')-V_{cl}(U)| \\
& +\theta(V')\theta(U)\ln|V-V_{cl}(U)| + \theta(U')\theta(V)\ln|V_{cl}(U')-V| \ (34)
\end{aligned}
$$

The first two terms give the unscattered propagator in the regions $U < 0$ all V's and $V > 0$ late U's. The second line describes the U-part of the scattered field configurations as in eq. (23). The third term encodes the correlations set up on $U = -\infty$ between positive and negative V's which have been split away from each other owing to the asymptotic behavior of the trajectory at late times. In addition to these unusual correlations, eq. (34) exhibits also clearly the *absence* of correlations[3] between positive and negative U's.

In order to understand the particle content of both aspects, it is very useful to work with Rindler modes, eigen modes of $iV\partial_V = \nu$. This is because $V_{cl} = -1/a^2 U$ becomes static in Rindler coordinates given by

$$
\begin{aligned}
av_L &= \theta(-V)\ln(-aV) & av_R &= \theta(V)\ln(aV) \\
au_L &= \theta(U)\ln(aU) & au_R &= \theta(-U)\ln(-aU)
\end{aligned}
\tag{35}
$$

---

[3]This absence of correlations is directly related to the presence of the above mentioned singular transcients which encode an infinite energy. This double aspect was recently found in the context of black hole evaporation in 2-D dilatonic gravity, see ref. [32]. As suggested in this reference, the origin of this singular behavior can be imputed to the use of the background field approximation. This will be explicitly proven in Section 5 in the case of the accelerating mirror.

Indeed the trajectory reads $u_L = v_L$. Therefore the scattering conserves the Rindler energy $\nu = i\partial_v$ and the Bogoliubov transformation relating Rindler modes is diagonal in ν.

The V-part of the in-vacuum is vacuum with respect to the "Unruh"-modes[20]

$$
\begin{aligned}
\varphi_{\nu,V}(V) &= \alpha_\nu \varphi_{\nu,R,V} + \beta_\nu \varphi^*_{\nu,L,V} \\
\varphi_{-\nu,V}(V) &= \alpha_\nu \varphi_{\nu,L,V} + \beta_\nu \varphi^*_{\nu,R,V}
\end{aligned}
\tag{36}
$$

where $\nu > 0$, where the Bogoliubov coefficients satisfy $|\beta_\nu/\alpha_\nu|^2 = e^{-2\pi\nu/a}$ and where the Rindler modes are[31][18]

$$
\varphi_{\nu,L,V} = \theta(-V)\frac{e^{-i\nu v_L}}{4\pi\nu} \ , \quad \varphi_{\nu,R,V} = \theta(V)\frac{e^{-i\nu v_R}}{4\pi\nu}
\tag{37}
$$

The U-part of the in-vacuum, for $U < 0$, is a thermal state with temperature $a/2\pi$ of Rindler modes defined by

$$
\varphi_{\nu,R,U} = \theta(-U)\frac{e^{-i\nu u_R}}{4\pi\nu}
\tag{38}
$$

The in vacuum can be expressed in terms of the Rindler operators $a_{\nu,R,V}$, $a_{\nu,L,V}$ and $a_{\nu,R,U}$ associated with the Rindler modes together with the corresponding b operators acting on the Rindler vacuum $|0,R\rangle_V|0,L\rangle_V|0,R\rangle_U|0,L\rangle_U$ as

$$
\begin{aligned}
|0,in\rangle &= \prod_\nu e^{arctanh(e^{-\pi\nu})\left[(a^\dagger_{\nu,R,V}b^\dagger_{\nu,L,V}+b^\dagger_{\nu,R,V}a^\dagger_{\nu,L,V})+(h.c.)\right]}|0,R\rangle_V|0,L\rangle_V \\
&\otimes \mathrm{Tr}_{(a_{\nu,L,U},\ b_{\nu,L,U})} e^{arctanh(e^{-\pi\nu})\left[(a^\dagger_{\nu,L,U}b^\dagger_{\nu,L,U}+b^\dagger_{\nu,R,U}a^\dagger_{\nu,L,U})+(h.c.)\right]}|0,R\rangle_U|0,L\rangle_U
\end{aligned}
\tag{39}
$$

where we have introduced the operators $a_{\nu,L,U}$ and $b_{\nu,L,U}$ to represent the thermal trace for the right U sector.

The U part of the out-vacuum is vacuum with respect to the out-modes given by the U version of the Unruh modes displayed in eq. (36), i.e. with U replacing V. The V part of the out-vacuum defined for $V > 0$ on $U = \infty$ can be represented as was represented the U-part of the in vacuum, i.e. by a thermal density matrix of Right Rindler modes given in eq. (37). Similarly, the out-vacuum can be expressed in terms of Rindler states as in eq. (39), with the substitution of V by U.

The scattering matrix in Rindler modes is trivial: it replaces $\varphi_{\nu,L,V}$ by $-\varphi_{\nu,L,U}$. Therefore, the scattered Unruh modes are

$$
\begin{aligned}
\varphi^{scat}_{\nu,V}(U,V) &= \alpha_\nu \varphi_{\nu,R,V} - \beta_\nu \varphi^*_{\nu,L,U} \\
&= \alpha_\nu \varphi_{\nu,R,V} - \beta_\nu(\alpha_\nu \varphi^*_{-\nu,U} - \beta_\nu \varphi_{\nu,U}) \\
\varphi^{scat}_{-\nu,V}(U,V) &= -\alpha_\nu \varphi_{\nu,L,U} + \beta_\nu \varphi^*_{\nu,R,V} \\
&= -\alpha_\nu(\alpha_\nu \varphi_{-\nu,U} - \beta_\nu \varphi^*_{\nu,U}) + \beta_\nu \varphi^*_{\nu,R,V}
\end{aligned}
\tag{40}
$$

In the second equalities we have expressed the Rindler U modes in the r.h.s. in terms of Unruh modes.

From eq. (40), one sees that the "partner", see eq. (30), of a particle described by a wave packet of $\varphi_{-\nu,U}$ has two branches. One reaches $V = \infty$ and is described by a superposition of $\varphi^*_{\nu,U}$ modes and the other one is a V mode which reaches $U = \infty$. Very important is the fact that it is described by a Rindler mode $\varphi^*_{\nu,R,V}$ since it exists only V. Therefore this part of the partner wave function is not a Minkowski quantum and we have not pair production of Minkowski quanta. See refs. [33][34] for a discussion of the correlations between asymptotic quanta and "vacuum configurations". I put quotation marks to emphasize the unusual aspects of these vacuum configurations since they are not expressible in terms of Minkowski modes globally defined. In order to palliate this cumbersome situation it is appropriate to decouple adiabatically, i.e. with respect to a $1/a$ proper time lapse, the mirror from the field configurations for asymptotic times $t \to \pm\infty$. Then, the asymptotic field configurations can be safely and correctly decomposed in terms of globally defined Minkowski modes both in the past, on $V = -\infty$ all U's, and the future, on $U = \infty$ all V's. It is very simple to perform this adiabatic decoupling in our case: it suffices to rewrite the V part of the scattered modes in terms of Unruh modes, eq. (36) which are globally defined. The adiabaticity is precisely what legitimates this simple procedure[13]. Thus the operators $a_{\nu,L,V}$ which were introduced only to represent, through a trace, the V part of the out vacuum as a thermal distribution of R quanta, are now treated on the same footing as the operators $a_{\nu,R,V}$ and truly represent the field configurations on the other side of the mirror. In terms of the Unruh V modes, the relation between the scattered modes and the out-modes globally defined are, see eqs. (40),

$$
\begin{aligned}
\varphi^{scat}_{\nu,V} &= (1 + \beta_\nu^2 T_\nu^*)\varphi_{\nu,V} - \alpha_\nu\beta_\nu T_\nu^* \varphi^*_{-\nu,V} + \alpha_\nu\beta_\nu T_\nu^* \varphi^*_{-\nu,U} - \beta_\nu^2 T_\nu^* \varphi_{\nu,U} \\
\varphi^{scat}_{-\nu,V} &= (1 - \alpha_\nu^2 T_\nu)\varphi_{-\nu,V} + \alpha_\nu\beta_\nu T_\nu \varphi^*_{\nu,V} - \alpha_\nu\beta_\nu T_\nu \varphi^*_{\nu,U} + \alpha_\nu^2 T_\nu \varphi_{-\nu,U}
\end{aligned}
\tag{41}
$$

together with the equations for the scattered U-modes given by the same equations with U and V interchanged. Indeed if one decouples the field configurations at $t = \infty$, the U modes are also scattered. We have introduced the complex coefficients T_ν to generalize to the case of partially reflecting mirrors. The coefficients T_ν satisfy the unitary relation: $\mathrm{Re}T_\nu = |T_\nu|^2$. Total reflection is given by $T_\nu = 1$. Of course for $T_\nu = 0$, there is no production since there is no scattering. One has now a "normal" pair creation phenomenon in which the asymptotic quanta are all free Minkowski quanta. The pairs are made out of U-U, U-V and V-V quanta. Therefore one sees that the adiabatic decoupling has replaced the correlations between Minkowski quanta and vacuum field configurations[33] by pair production occurring on both sides of the mirror. It should be noted that the decoupling has been introduced here in a rather *ad hoc* way. Nevertheless, this procedure will be justified in the next sections since we shall see that it will come up automatically, in perturbation theory, when we shall compute the S matrix elements. Note that one can also refuse to switch on and off asymptotically the coupling to the mirror. In that case, one has to analyze the scattering in terms of asymptotic states which are not the free Minkowski modes but which are instead characterized by a "final state interaction" with the mirror.

The adiabatic decoupling is already justified by following remark. The partial scattering from the mirror, eq. (41), is very similar to the scattering of light like quanta

induced by Unruh effect[20], i.e. the fact that the inner degrees of freedom of an accelerated system thermalize with temperature $a/2\pi$. Indeed one can pass continuously from one case to the other. Take for instance the model of the oscillator introduced by Raine, Sciama and Grove[35], see also [36]. In that model the oscillator is coupled to the derivative of the field by the hamiltonian $H_{int} = e \int d\tau i[\dot{\phi}(\tau) - \dot{\phi}^\dagger(\tau)](Ae^{-i\mu\tau} + A^\dagger e^{i\mu\tau})$. A ($A^\dagger$) is the lowering (raising) operator and μ is the energy gap between two states. Unruh's analysis[20] shows that this accelerated system thermalizes in Minkowski vacuum with temperature $a/2\pi$, i.e. that the equilibrium probabilities to find the oscillator in its nth excited state satisfy $P_n = e^{-2\pi n\mu/a}P_{gr.}$, where $P_{gr.}$ is the probability to find the oscillator in its ground state. Upon eliminating the oscillator operators, one can express the scattered Rindler modes in terms of the out Rindler modes. One obtains[13]

$$\varphi_{\nu,L,U}^{scat} = \varphi_{\nu,L,U}(1 + i\frac{e}{2}\psi_\nu) + (i\frac{e}{2}\psi_\nu)\varphi_{\nu,L,V} \tag{42}$$

where ψ_ν is the oscillator's propagator given by

$$\psi_\nu = \frac{e\nu}{\mu^2 - \nu^2 - ie^2\nu/2} \tag{43}$$

Therefore $T_\nu = ie\psi_\nu/2$. Furthermore upon taking the limit $e \to \infty$, one obtains total reflection.

By virtue of this mapping, all properties of $T_{\mu\nu}$ including the conditional value of $T_{\mu\nu}$, eq. (31), are in strict analogy with the properties obtained from Unruh effect. We therefore refer to [13] for a detailed analysis of the the fluctuating properties of $T_{\mu\nu}$. Here, we shall only present an interesting link between the fluxes emitted by an accelerated oscillator and by the accelerated mirror which sheds light on the nature of the mirror when viewed as a quantum system.

In the low coupling limit, i.e for $e^2 << \mu$, the norm of the oscillator's propagator ψ_ν tends to $\delta(\mu - \nu) + \delta(\mu + \nu)$. Then, once thermal equilibrium is achieved, the mean flux emitted by the oscillator is, see [21][22][35][13],

$$\begin{aligned}
\langle T_{UU}\rangle_\mu^{atom} &= \theta(U)\, 2e^2\mu \int_0^\infty d\lambda \int_0^\infty d\lambda' \frac{\sqrt{\lambda\lambda'}}{2\pi} \\
&\quad \frac{1}{1 - e^{-2\pi\mu/a}}\left[e^{-i(\lambda-\lambda')U}\beta_{\mu,\lambda}^{L*}\beta_{\mu,\lambda'}^{L} - \text{Re}\left(e^{-i(\lambda+\lambda')U}\alpha_{\mu,\lambda}^{L}\beta_{\mu,\lambda'}^{L*}\right)\right] \\
&\quad + \frac{1}{e^{2\pi\mu/a} - 1}\left[e^{-i(\lambda-\lambda')U}\alpha_{\mu,\lambda}^{L}\alpha_{\mu,\lambda'}^{L*} - \text{Re}\left(e^{-i(\lambda+\lambda')U}\beta_{\mu,\lambda}^{L*}\alpha_{\mu,\lambda'}^{L}\right)\right] \quad (44)
\end{aligned}$$

where $\beta_{\mu,\lambda'}^{L}$ and $\alpha_{\mu,\lambda}^{L}$ are the Bogoliubov coefficients between Rindler modes of frequency μ and the Minkowski modes of frequency λ. The first line comes from the excitation process and the second line from the deexcitation. The thermal factors multiplying the brackets are respectively the probabilities $P_{gr.}$ and P_1. The overall factor μ comes from the derivative coupling and the relativistic normalization of the modes.

Thus upon summing $\langle T_{UU}\rangle_\mu^{atom}/\mu$ over all μ's, one obtains e^2 times the flux reflected by the mirror reexpressed in terms of Unruh modes, eq. (36), instead of the usual Minkowski modes as done in eq. (22). Indeed in that equation, the sum over all positive

ω can be replaced by any other complete set of modes of positive in frequency. Therefore, the mirror acts as a collection of weakly coupled (to the time derivative of the field) oscillators whose density of probability of finding the frequency μ is given by $d\mu/\mu$.

It is now obvious that the (mean) total number of Minkowski quanta emitted by the mirror increases with the lapse of proper time during which the mirror interacts with the field[13], since each excitation or deexcitation process leads to the production of a Minkowski quantum in spite of the fact that the *local* flux, $\langle T_{UU} \rangle_\mu^{atom}$, vanishes for all μ in the intermediate region. In Section 5, we shall see that the vanishing character of $\langle T_{UU} \rangle$ as well as its singular behavior at $U = 0$ are both directly attributable to the classical and unaffected character of the mirror's trajectory. Indeed, upon taking into account the recoils of the mirror, we shall see that the local repartition of the energy content of the emitted quanta is completely modified after a certain lapse of proper time even though neither the mean number of quanta emitted nor their energy is changed.

4   The Dynamical mirror

We shall introduce and analyze a dynamical version of the DF model in the simplest case, that of the inertial mirror. A dynamical non inertial mirror will be considered in the next section. We shall proceed in two steps. First we shall introduce a self interacting model without dynamics. The reasons for this intermediate step are the following:
1) The intermediate model gives back the DF model upon resuming the Born series.
2) The perturbative matrix elements of the intermediate model coincide with the large mass limit of the matrix elements of dynamical model we shall discuss afterwards.

Therefore, the dynamical model is related to the DF model by a well defined double procedure: by a limit of large mass and by resummation of perturbative effects. In addition to the large mass limit, we shall also describe the other limit where the energy of the incident quantum approaches the mirror's mass. In this case, we shall see that the mass of the mirror acts as a cut off for the reflected energy.

From the equality of the matrix elements in the large mass limit, (point 2), it is tempting to conclude that one should obtain, in this limit, the same expression as the one obtained in the DF or the intermediate model whatever is the quantity calculated. This is not the case. The reason is that the dynamical model contains a third dimensionful quantity, in addition to the frequency of the scattered light and the mass of the mirror, namely the spread of the wave function which characterizes the position of the mirror. Together with the momentum transfer to the mirror upon reflection, this spread intervenes in decoherence effects when the frequency of the scattered light is comparable or bigger than its inverse but nevertheless much smaller than the rest mass of the mirror. To describe this decoherence we shall study the mean scattered flux when the incident radiation state is described by a pure state which contains states with different particle number. In that case, the mean flux can be decomposed into two terms as in eqs. (11, 22). We shall see that each term is differently affected by the recoils and that these modifications lead to decoherence effects.

We start by analyzing the intermediate model in order to prove point 1). The equa-

tion which replace eqs. (1, 2) in the inertial case and which defines the intermediate model is

$$\left[\partial_t^2 - \partial_z^2\right]\phi(t,z) = g\delta(z)\, 2i\partial_t\phi(t,z) \tag{45}$$

where g is a dimensionless coupling constant and $\delta(z)$ is a δ of Dirac which specifies that the scatterer sits for all times on $z = 0$.

Our goal is to prove that in the large (compare to 1) g limit, the scattering of light induced by the r.h.s of eq. (45) gives back the DF model, i.e. gives back the coefficients $\gamma_{\omega,\lambda}$ defined in eqs. (7, 8). This is an easy task since eq. (45) is linear in ϕ. Therefore, in classical as well as in quantum mechanics, the solution can be expressed as

$$\phi(t,z) = \phi^{in}(t,z) + \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dz' G^{ret}(t',z';t,z)g\delta(z')\, 2i\partial_t\phi(t',z') \tag{46}$$

where ϕ^{in} satisfies the free, $g = 0$, d'Alembertian and where G^{ret} is the retarded Green function. In Fourier decomposition is it given by

$$G^{ret}(t',z';t,z) = \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} dk \frac{1}{4\pi^2} \frac{e^{i\omega(t'-t)-ik(z'-z)}}{(\omega+i\epsilon)^2 - k^2} \quad (= 0 \text{ for } t' > t) \tag{47}$$

Since the location of the mirror is static it is appropriate to work at fixed energy[25]. One defines

$$\phi_\omega(z) = \int_{-\infty}^{\infty} \frac{dt}{2\pi}\phi(t,z)e^{i\omega t} \tag{48}$$

In terms of these Fourier components, eq. (46) becomes

$$\begin{aligned}
\phi_\omega(z) &= \phi_\omega^{in}(z) + ig\phi_\omega(z=0)(2i\omega)\int \frac{dk}{2\pi} \frac{e^{ikz}}{(\omega+i\epsilon)^2 - k^2} \\
&= \phi_\omega^{in}(z) - ig\phi_\omega(0)e^{-i\omega|z|} \tag{49}
\end{aligned}$$

Similarly one can express the Heisenberg (interacting) field ϕ in terms of the out-field ϕ^{out} and the *advanced* Green function given by eq. (47) with $\epsilon = 0^-$. One gets

$$\phi_\omega(z) = \phi_\omega^{out}(z) + ig\phi_\omega(0)e^{i\omega|z|} \tag{50}$$

Therefore one can eliminate the interacting fields $\phi_\omega(z)$ and $\phi_\omega(z = 0)$ and one obtains

$$\phi_\omega^{out}(z) = \phi_\omega^{in}(z) + ig\left(\frac{1}{1-ig}\right)\phi_\omega^{in}(z=0)\, 2\cos(\omega z) \tag{51}$$

By expressing the operator $\phi_\omega^{in}(z)$ is terms of the usual Minkowski operators $a_{\omega,i}$ where $i = U$ or V and $\omega = |k|$ is the energy, one gets

$$a_{\omega,U}^{out} = a_{\omega,U}^{in}\left(1 + \frac{ig}{1-ig}\right) + a_{\omega,V}^{in}\left(\frac{ig}{1-ig}\right) \tag{52}$$

Thus for $g \to \infty$, one obtains pure reflection for an inertial mirror, i.e. $a_{\omega,U}^{out} = -a_{\omega,V}^{in}$ or $\gamma(\omega,\lambda) = -\delta(\omega-\lambda)$ in eq. (8). We have therefore proven point 1) given above. Had we

taken $g\delta(z)\phi$ for the r.h.s. of eq. (45), we would have got, as in ref. [25], a frequency dependent coefficient.

In preparation for point 2), we compute the perturbative matrix elements of this intermediated model. In the interacting representation, one has

$$S_{\omega,\lambda} = \langle 0|a_{\lambda,j}\, S\, a^\dagger_{\omega,i}|0\rangle = \langle 0|a_{\lambda,j}\, \mathrm{T}e^{-i\int dt H_{int}}\, a^\dagger_{\omega,i}|0\rangle \tag{53}$$

where $|0\rangle$ is Minkowski vacuum, T is the time ordered product and H_{int} is the interaction hamiltonian given by

$$H_{int} = g\int_{-\infty}^{\infty} dz\delta(z)\phi^\dagger i\overleftrightarrow{\partial_t}\phi = g\int_{-\infty}^{\infty} dz\delta(z)J_t(t,z) \tag{54}$$

Using this hamiltonian, the Hamilton equations lead to eq. (45). To first order in g, the matrix element is

$$S_{\omega,\lambda} = \delta(\omega-\lambda)\delta_{ij} - ig\int_{-\infty}^{\infty} dt\langle\lambda,j|\phi^\dagger i\overleftrightarrow{\partial_t}\phi|\omega,i\rangle = \delta(\omega-\lambda)\delta_{ij} - ig\,\delta(\omega-\lambda) \tag{55}$$

where we have written momentum conservation as energy conservation times the δ_{ij} of Kronecker between U and V sectors. Notice that the second term, linear in g, expresses only energy conservation as it should do. Indeed, to first order in g, it corresponds to the result of eq. (52) in the interacting picture.

Notice also that one can easily generalize this scattering to non inertial trajectories. It suffices to replace the interaction hamiltonian, eq. (54), by

$$H_{int} = g\int_{-\infty}^{\infty} dz\delta(z_{cl}(t) - z)\dot{x}^\mu_{cl}(t)J_\mu(t,z) \tag{56}$$

where $\dot{x}^\mu_{cl}(t) = dx^\mu_{cl}(t)/dt$ is the tangent vector of the classical trajectory $z = z_{cl}(t)$ and J_μ is the current operator. In this non inertial case, to first order in g, one verifies that the matrix elements $S_{\omega,\lambda}$ identically give the $\gamma_{\omega,\lambda}$ coefficients of the DF model, see eq. (8), since these later are given in terms of matrix elements of the current, see eq. (4). Finally one can also generalize this model to smeared off δ functions. One then finds that the matrix elements decrease when the incident frequency ω is higher than the frequency content of this smeared function.

We now define the inertial version of our dynamical model for the mirror. The mirror is described by a charged scalar massif field ψ of mass M coupled to the massless field ϕ by the following interaction hamiltonian

$$\tilde{H}_{int} = \tilde{g}\int_{-\infty}^{\infty} dz J^\mu_\psi(t,z)J^\phi_\mu(t,z) \tag{57}$$

This hamiltonian gives a four point interaction through the currents carried by ψ and ϕ. (This is why we choose to work with complex fields.) The Euler-Lagrange equations are

$$\begin{aligned}
\left[\partial_t^2 - \partial_z^2\right]\phi(t,z) &= \tilde{g}J^\mu_\psi(t,z)\, 2i\partial_\mu\phi(t,z) \\
\left[\partial_t^2 - \partial_z^2 + M^2\right]\psi(t,z) &= \tilde{g}J^\mu_\phi(t,z)\, 2i\partial_\mu\psi(t,z)
\end{aligned} \tag{58}$$

One can verify that in a naive "background field approximation" this model gives back the intermediate model. Indeed, in the large mass limit and upon neglecting recoils, the current carried by the heavy ψ field can be approximated by a $\dot{x}^\mu_{cl}(t)\delta(z_{cl}(t) - z)$ evaluated along its classical trajectory. In this approximation, if $\tilde{g} = g$, the hamiltonian $\tilde{H}_{int}$ gives back the hamiltonian of the intermediate model given in eq. (56). We shall not pursue in this way but on the contrary we shall work in a pure quantum mechanical framework.

Therefore we shall compute the perturbative S matrix elements induced by $\tilde{H}_{int}$. We work in the interacting representation. Then the annihilation and creation operators are the usual free Minkowski ones. The operators for the ϕ field have already been given. The operators for the ψ field are obtained by decomposing the free $\tilde{g} = 0$ solutions of eq. (58) in Fourier terms

$$\psi(t, z) = \int_{-\infty}^{\infty} dp \left[c_p \frac{e^{-i(\Omega_p t - pz)}}{\sqrt{4\pi\Omega_p}} + d_p^\dagger \frac{e^{i(\Omega_p t - pz)}}{\sqrt{4\pi\Omega_p}} \right] \tag{59}$$

where $\Omega_p^2 = p^2 + M^2$. The operators c_p annihilate the vacuum state $|0\rangle_{Mir}$.

The matrix element between an initial state containing an heavy particle of momentum p and a lightlike quantum of momentum k_ω and a final state of momenta p' and k_λ is

$$\tilde{S}(p; k_\omega; k_\lambda) = \langle\tilde{0}|c_{p'}a_{k_\lambda} \; \tilde{S} \; a_{k_\omega}^\dagger c_p^\dagger|\tilde{0}\rangle = \langle\tilde{0}|c_{p'}a_{k_\lambda} \; \mathrm{T}e^{-i\int dt\tilde{H}_{int}} \; a_{k_\omega}^\dagger c_p^\dagger|\tilde{0}\rangle \tag{60}$$

where $|\tilde{0}\rangle = |0\rangle_{Mir}|0\rangle$. To first order in $\tilde{g}$, $\tilde{S}(p; k_\omega; k_\lambda)$ is given by (compare with eq. (55))

$$\begin{aligned} \tilde{S}(p; k_\omega; k_\lambda) &= \delta(p' - p)\delta(\lambda - \omega)\delta_{ji} - i\tilde{g}\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dz\langle p'|\psi^\dagger i\overleftrightarrow{\partial}_\mu \psi|p\rangle\langle k_\lambda|\phi^\dagger i\overleftrightarrow{\partial}^\mu \phi|k_\omega\rangle \\ &= \delta(p' - p)\delta(k_\lambda - k_\omega) - i\tilde{g}\delta(p' - p - k_\omega + k_\lambda)\delta(\Omega_{p'} + \lambda - \Omega_p - \omega) \\ &\qquad \left[\frac{(\Omega_{p'} + \Omega_p)(\omega + \lambda) - (p' + p)(k_\omega + k_\lambda)}{4\sqrt{\Omega_{p'}\Omega_p\omega\lambda}} \right] \end{aligned} \tag{61}$$

The first δ of the linear term in $\tilde{g}$ expresses momentum conservation and the second one gives energy conservation. When the initial momentum p vanishes, this latter gives

$$\begin{aligned} \lambda &= \omega \quad \text{if} \quad i = j \\ \lambda &= \frac{\omega}{1 + 2\omega/M} \quad \text{if} \quad i \neq j \end{aligned} \tag{62}$$

As for the passive mirror, there is no energy transfer in the case of scattering which conserves U-ness or V-ness; only a phase is introduced upon scattering. On the contrary, in the reflection case, one recovers energy conservation only for the photons whose energy satisfies $\omega/M << 1$. In that case the coefficient in bracket in eq. (61) is unity for all such small ω and λ. When $\omega/M << 1$ but not negligible, the first correction to the energy relation between λ and ω, eq. (62), and to the bracket of eq. (61) is the non relativistic energy $(2\omega)^2/2M$ of the mirror. Therefore we have proven point 2) since, in

the limit $M \to \infty$ at fixed ω, λ and p, $\tilde{S}(p; k_\omega; k_\lambda) = S(k_\omega; k_\lambda)$, the matrix element of eq. (55).

On the contrary, when $\omega/M \to \infty$, one finds $\lambda \to M/2$. Thus, the finiteness of the mass of the mirror acts as a U.V. cut off for the reflected frequency and the discrepancy with the DF model is total. In that respect we mention ref. [37] wherein it is shown how an U.V. cutoff on the reflection coefficients eliminates the divergent expressions of the Casimir effect (and therefore the necessity of the regularization and the renormalization procedure) since these frequencies no longer contribute to the Casimir force. Notice also that both the correction term $(2\omega)^2/2M$ for small ω/M and the asymptotic behavior $\lambda \to M/2$ for large ω/M are quantum mechanical in nature: reinstating $\hbar$ one has: $\lambda = \omega - \hbar(2\omega)^2/2M$ and $\lambda \to \hbar M/2$. In classical mechanics these terms encode the nonlinearity of the theory since the scattered frequency is now a function of the *norm* of the incident wave.

In view of the fact that in the limit $M \to \infty$ at fixed ω, λ, p, one has $\tilde{S}(p; k_\omega; k_\lambda) = S(k_\omega; k_\lambda)$, it is tempting to conclude that the results evaluated in the dynamical model would coincide, in *all situations*, with the ones obtained in this no-recoil model. This is not the case. There are circumstances in which interferences induced by momentum transfer to the mirror upon reflection ruin the equality of the results. Indeed being related to momentum transfer, the interferences subsist in the limit $M \to \infty$ at fixed ω, λ, p. Furthermore, they induce decoherence. To show how and why this happens, we shall now study the properties of the scattered flux.

The scattered flux

Our goal is to show that even in the limit $M \to \infty$, in certain circumstances, expectation values of local operators do not coincide with the corresponding expectation values evaluated in the no-recoil model because additional phase factors induced by recoils arise in the expectation values. To describe the role of these factors we shall compute the local properties of the reflected flux when the radiation state contains states with different particle number. In that case, the mean flux possesses two terms as in eqs. (11, 22). Then we shall see clearly the different consequencies of the recoils on each term. Similar effects will be found in the next section upon computing the properties of the flux spontaneously produced by a non-inertial mirror. In both cases, when the momentum transfers are comparable to the inverse spread of the wave function of the mirror, the second term of the decomposition of the mean flux vanishes.

We work in the limit $\omega/M \to 0$ and $p/M \to 0$. Then the energy of the radiation is exactly conserved, see eq. (62), and the mirror has no energy: its wave function does not spread in time but it still absorbs momentum upon reflecting a quantum. To exhibit how recoils lead to decoherence, we shall analyze the scattering of the following radiation state

$$|\chi\rangle = A|0\rangle + B \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \; f_{\omega_1} g_{\omega_2} a_{\omega_1, V}^\dagger a_{\omega_2, V}^\dagger |0\rangle = A|0\rangle + B|2\rangle \qquad (63)$$

where A and B are coefficients such that $\langle \chi | \chi \rangle = 1$ and where f_ω and g_ω describe two normalized wave packets which have non overlapping frequency range. (This requirement

simplifies the following equations.) The reason for having chosen $|\chi\rangle$ is that it contains states whose particle number differ by two, hence there will be a contribution to the mean flux arising from the "interfering" second term of eq. (22). We shall say that the radiation states decohere when the scattered state of the coupled system radiation+mirror is such that this interfering term vanishes owing to the recoils induced by the reflection of the quanta building this flux.

Before scattering, the normalized wave function of the mirror is

$$|Mir\rangle = \int_{-\infty}^{\infty} dp\ h_p c_p^\dagger |0\rangle_{Mir} \tag{64}$$

where the momentum profile h_p is $e^{-p^2/2\sigma^2}/(\pi\sigma^2)^{1/4}$. We do not assume that the frequency content of f_ω and g_ω is negligible with respect to σ, in which case one would have a mirror with a well defined position with respect to $1/\omega$. On the contrary, we shall see that decoherence occurs when the characteristic frequencies in f or g are comparable or greather than σ.

The initial state of the total system is thus $|in\rangle = |Mir\rangle|\chi\rangle$. Before studying the scattering, we analyze the properties of the initial flux. The total energy of the radiation, before scattering, is

$$
\begin{aligned}
\langle H_V \rangle &= \langle in| \int_0^\infty d\omega \omega a_{\omega,V}^\dagger a_{\omega,V} |in\rangle = \langle \chi| \int_0^\infty d\omega \omega a_{\omega,V}^\dagger a_{\omega,V} |\chi\rangle \\
&= |B|^2 \left[\int_0^\infty d\omega \omega |f_\omega|^2 + \int_0^\infty d\omega \omega |g_\omega|^2 \right]
\end{aligned}
\tag{65}
$$

since $\langle Mir|Mir\rangle = 1$. The mean energy is thus the sum of the mean energies of each initial quantum times the probability to find it.

The mean initial energy density is not so simple. It is given by

$$
\begin{aligned}
\langle T_{VV} \rangle &= \langle in|\partial_V \phi^\dagger \partial_V \phi + \partial_V \phi \partial_V \phi^\dagger |in\rangle = \langle \chi|\partial_V \phi^\dagger \partial_V \phi + \partial_V \phi \partial_V \phi^\dagger |\chi\rangle \\
&= |B|^2 \langle 2|\partial_V \phi^\dagger \partial_V \phi + \partial_V \phi \partial_V \phi^\dagger |2\rangle + 2\mathrm{Re}\left[A^* B \langle 0|\partial_V \phi^\dagger \partial_V \phi |2\rangle \right]
\end{aligned}
\tag{66}
$$

where $|2\rangle$ designates the second term of eq. (63). The first term of eq. (66) comes from the diagonal part of the operator T_{VV}. It is given by

$$|B|^2 \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{\sqrt{\omega_1 \omega_2}}{2\pi} e^{-i(\omega_1-\omega_2)V} \left[f_{\omega_1} f_{\omega_2}^* + g_{\omega_1} g_{\omega_2}^* \right] \tag{67}$$

This term is thus the sum of the mean fluxes carried by each wave function times the probability to find the particles. Each contribution corresponds exactly to the first term of eqs. (11, 22). (There is no crossing term in $g_{\omega_1}^* f_{\omega_2}$ because we have imposed that the frequency ranges of f and g do not overlap.)

The second term of eq. (66) comes from interferences between states whose particle number differ by two. It reads

$$\int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{-\sqrt{\omega_1 \omega_2}}{2\pi} \mathrm{Re}\left[e^{-i(\omega_1+\omega_2)V} \left(A^* B f_{\omega_1} g_{\omega_2} \right) \right] \tag{68}$$

As for the second terms of eqs. (11, 22), when integrated over all V's, it vanishes identically. Notice that it depends also of the phase of A^*B, i.e. the relative phase between the vacuum and the two particle state. We shall now prove that this term is sensitive to the above mentioned additional phases induced by the recoils. To this end, we compute the scattered state and the properties of the scattered flux. We shall then compare these properties with the properties before scattering.

To order $\tilde{g}^2$, the scattered state on $t = \infty$ has the following structure,

$$\tilde{S}|in\rangle = |in\rangle - i\tilde{g}|Mir', 1scat\rangle - \tilde{g}^2|Mir'', 2scat\rangle \tag{69}$$

where $|Mir', 1scat\rangle$ designates the state of the system on which the interaction hamiltonian, eq. (57), has acted only once. In that state only one particle has been scattered. The state $|Mir'', 2scat\rangle$ is obtained by acting twice with this hamiltonian. The relevant part of this state is that part in which both particles have been reflected. Indeed the other parts will not contribute to the following expressions, at least to order g^2. The explicit expressions of these states are easily obtained in the Golden Rule approximation since in that case, it is legitimate to neglect the time ordering appearing in the $\tilde{g}^2$ term, see [13]. They will not be explicitized here. What interests us is the structure of the scattered flux.

To order $\tilde{g}^2$, the reflected mean energy is

$$\begin{aligned}
\langle H_U^{scat} \rangle &= \langle in|\tilde{S}^\dagger \, H_U \, \tilde{S}|in\rangle = \tilde{g}^2 \langle Mir', 1scat| \int_0^\infty d\lambda\lambda a_{\lambda,U}^\dagger a_{\lambda,U}|Mir', 1scat\rangle \\
&= \tilde{g}^2|B|^2 \left[\int_0^\infty d\lambda\lambda|f_\lambda|^2 + \int_0^\infty d\lambda\lambda|g_\lambda|^2 \right]
\end{aligned} \tag{70}$$

Note that the twice scattered state $|Mir'', 2scat\rangle$ does not contribute to $\langle H_U^{scat}\rangle$. Furthermore, since H_U is diagonal in λ, only diagonal terms in p' contribute (where p' is the scattered momentum of the mirror: $p' = p - 2\omega$, see eq. (61)). Thus the scattered mirror wave function factorizes out. To obtain the third equality, we have used the kinematical conditions: $\omega_i << M$ and $p << M$ which give $\lambda_i = \omega_i$. Thus we see that the reflected energy is simply the initial energy, eq. (65), times $\tilde{g}^2$, the probability to be reflected by the mirror. This is true no matter how big or small is σ, the spread of momentum of the initial wave packet of the mirror, eq. (64).

The reflected local flux of energy is more complicated and does depend on σ. To order $\tilde{g}^2$, one finds

$$\begin{aligned}
\langle T_{UU}^{scat} \rangle &= \tilde{g}^2|B|^2\langle Mir', 1scat|\partial_U\phi^\dagger\partial_U\phi + \partial_U\phi\partial_U\phi^\dagger|Mir', 1scat\rangle \\
&\quad -\tilde{g}^2 \, 2\mathrm{Re}\left[A^*B\langle Mir|\langle 0|\partial_U\phi^\dagger\partial_U\phi|Mir'', 2scat\rangle \right]
\end{aligned} \tag{71}$$

Straightforward algebra gives both terms. Before giving the resulting expressions it is worthwhile to make the following observation. In the second term, upon evaluating the matrix element of T_{UU}, one finds that the overlap the mirror's unscattered state with the twice scattered one gives: $\int dp h_p^* h_{p-2\omega_1-2\omega_2} = e^{-2(\omega_1+\omega_2)^2/\sigma^2}$. Therefore if the characteristic frequencies of f and g are bigger than σ, this term vanishes. The former interferences between $|0\rangle$ and $|2\rangle$ have been washed out by the momentum transfers

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to the mirror. In position terms this washing out may be understood as follows. The interfering pattern of the incident flux encoded in eq. (68) is characterized by the inverse mean frequency $1/\omega$. Upon scattering, it is smeared out over the inverse frequency of the mirror: $1/\sigma$ and therefore it vanishes since integrated over *all U*'s it does so. This simple averaging seems to be directly attributable to the broad character of the initial wave of the mirror and would correspond to the scattering from a smeared out passive mirror, i.e. by the hamiltonian eq. (54) with $\delta(z)$ replaced by a $d(z) = \int dp e^{ipz} h_p$. This is an overhasty conclusion since for $\omega >> 1/\sigma$, the passive mirror is completely transparent.

Explicit expressions for both terms of eq. (71) are listed below. The first one is

$$\tilde{g}^2 |B|^2 \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{\sqrt{\omega_1 \omega_2}}{2\pi} e^{-i(\omega_1 - \omega_2)U} \left(f_{\omega_1} f_{\omega_2}^* + g_{\omega_1} g_{\omega_2}^* \right) \int dp\, h_{p-2\omega_2}^* h_{p-2\omega_1} \qquad (72)$$

As in eq. (67), we have used the non-overlapping character of the frequency ranges of f and g. Notice how the overlap of the two scattered states of the mirror enters into the integrant. When $\sigma << \omega$, its role is to spread out the individual beams of energy on scales given by $1/\sigma$. The second term of eq. (71) is

$$- \tilde{g}^2 \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{-\sqrt{\omega_1 \omega_2}}{2\pi} \text{Re} \left[e^{-i(\omega_1 + \omega_2)U} \left(A^* B f_{\omega_1} g_{\omega_2} \int dp\, h_p^* h_{p-2\omega_1-2\omega_2} \right) \right] \qquad (73)$$

compare with eq. (68). For obtaining this result we have neglected the time ordering appearing to this order in $\tilde{g}$. It is a legitimate simplification in our case where $\omega/M << 1$. As already mentioned the overlap between the two states of the mirror vanishes when $\sigma << \omega$.

It is worthwhile to point out that in the limit $\sigma >> \omega$ these expressions are equal (up to a sign) to the ones obtained before scattering. Indeed eq. (72) coincides with eq. (67) and eq. (73) is the opposite of eq. (68). This sign flip comes from two developments of $\tilde{S}^\dagger T_{UU} \tilde{S}$ to order $\tilde{g}^2$. Indeed, for small reflection coefficient, the phase shift of the reflected wave is $\pi/2$ whereas in the large reflection limit, i.e. in the DF case, the shift is π, see eq. (52). It should also be pointed out that the relations between eqs. (72, 73) and eqs. (67, 68) are valid only if $\sqrt{\sigma}$ remains much smaller than the rest mass M. Indeed if this latter condition is not satisfied, one would find Doppler effects and more importantly, one should take into account the phases introduced by the kinetic energy of the mirror. This latter effect cannot be ignored when the time interval becomes comparable to M/σ^2. Therefore the DF results have a validity which is limited in both frequency ranges and time intervals. This latter condition on a maximal interval of time will reappear in the next section for the accelerated mirror with greater strength owing to the relativistic character of that trajectory. Furthermore, it would be interesting to extend those results to higher order in $\tilde{g}$. This might lead to stronger restrictions.

Before analyzing the non-inertial model, we briefly discuss the correlations between the scattered momentum of the mirror and the radiation state. We just saw how these correlations affect the mean value of the scattered flux. To analyze the individual correlations, we consider the conditional value of the flux when one knows the final position of the mirror, see eq. (31) for the definition of the conditional value of T_{UU}. In this case

the conditional flux is given by

$$\langle T_{UU}^{scat} \rangle_{\Pi_z} = \frac{\langle in| \tilde{S}^\dagger \ \Pi_z T_{UU} \ \tilde{S}|in\rangle}{\langle in|\tilde{S}^\dagger \ \Pi_z \ \tilde{S}|in\rangle} \tag{74}$$

where $\Pi_z = \mathrm{I}_\phi |z\rangle\langle z|$ is the projector which specifies that the final state of the mirror is given by $\int dp e^{ipz} c_p^\dagger |0\rangle_{Mir} = |z\rangle$ and which gives no restriction on the radiation state.

To order $\tilde{g}^2$, $\langle T_{UU}^{scat} \rangle_{\Pi_z}$ is again given by two terms

$$\langle T_{UU}^{scat} \rangle_{\Pi_z} = \tilde{g}^2 |B|^2 \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{\sqrt{\omega_1 \omega_2}}{2\pi} e^{-i(\omega_1 - \omega_2)(U-2z)} \left[f_{\omega_1} f_{\omega_2}^* + g_{\omega_1} g_{\omega_2}^* \right]/D$$

$$-\tilde{g}^2 \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{-\sqrt{\omega_1 \omega_2}}{2\pi} \mathrm{Re} \left[e^{-i(\omega_1 + \omega_2)(U-2z)} \left(A^* B f_{\omega_1} g_{\omega_2} \right) \right]/D \tag{75}$$

where $D = \langle in|\tilde{S}^\dagger \ \Pi_z \ \tilde{S}|in\rangle$ is the probability to find the mirror at z for $t = \infty$. To this order in $\tilde{g}$, it is evaluated without scattering and thus equal to $|\int dp e^{ipz} h_p|^2$. Notice that z should not be too large otherwise one obtains $D < \tilde{g}^2$ which invalidates eq. (75). When $D >> \tilde{g}^2$, one sees that the fact to have inserted the projector Π_z restores the initial energy distribution, see eqs. (67, 68), up to the sign flip between the first and the second term, c.f. the discussion above eq. (74). Notice also that the final specification of the position of the mirror introduces the phase shifts $e^{2i\omega z}$, as if the *initial* state of the mirror was $|z\rangle$.

Therefore, the decoherence obtained in eq. (71) upon computing the mean flux, i.e. the vanishing of the interfering second term given in eq. (73), is "removed" when one knows simultaneously the final state of the mirror *and* the value of the flux in this particular channel. It is the access to these non local correlations which reveals the "purity" of the state of the whole system since the momentum transfers to the mirror has mixed the states of radiation with the mirror's state and therefore has destroyed the purity of the initial radiation, see eq. (63) and eq. (68).

5 The Non Inertial Dynamical Mirror

We shall generalize the previous results to non-inertial trajectories, in particular to the uniformly accelerated mirror.

Since the mirror is dynamical, the only way to make it accelerate is to introduce an external field. The simplest case is to consider an electro-magnetic field coupled to the massive ψ field only. In that case, the dynamical equations (58) become

$$\left[\partial_t^2 - \partial_z^2\right] \phi(t,z) = \tilde{g} J_\psi^{\mu,A}(t,z) \ 2i\partial_\mu \phi(t,z)$$

$$\left[(\partial_t - iA_t(t,z))^2 - (\partial_z - iA_z(t,z))^2 + M^2\right] \psi(t,z) = \tilde{g} J_\phi^\mu(t,z) \ 2(i\partial_\mu - A_\mu)\psi(t,z) \tag{76}$$

where the current of the ψ field is now

$$J_\psi^{\mu,A}(t,z) = \psi^\dagger(t,z) i \overleftrightarrow{\mathcal{D}}_\mu \psi(t,z) = \psi^\dagger(t,z) \left[i\overleftrightarrow{\partial}_\mu - 2A_\mu(t,z)\right] \psi(t,z) \tag{77}$$

One can proceed as before and compute perturbatively the matrix elements of $\tilde{S}$. One solves the free d'Alembertian for ϕ. For ψ one solves the Klein Gordon equation in the presence of the A_μ field, perturbatively or non-perturbatively. It is worthwhile to point out that the perturbative treatment, in A_μ, needs not give the same amplitudes for pair creation as the ones obtained non-perturbatively, see refs. [38][39] in that respect.

We now specialize to the case of the constant electric field. We proceed as follows. First we obtain the non-perturbative solutions of the Klein Gordon equation in the electric field. Then we compute perturbatively (in $\tilde{g}$) the scattering matix elements. We show that to first order both in $\tilde{g}$ and in the momentum transfer, these amplitudes are identical to the ones obtained without recoil in the DF model, see eqs. (8). Nevertheless a difference in the interpretation occurs. What replaces the β coefficient is now directly interpretable as the amplitude of probability to make a pair of Minkowski quanta, and no longer as an overlap between positive and negative frequency solutions.

We then take into account recoil effects, i.e. higher orders in the momentum transfer. As in ref. [16], we shall see that the emissions of forward quanta decohere due to the recoils induced by these emissions. Contrary to what happens in the previous section, the decoherence is now inevitable owing to the properties of the spontaneously emitted quanta. We insist on this fact. Since the production is spontaneous and has its properties determined by the trajectory of the mirror, one can no longer intervene from the outset and specify to work in a given regime. I believe that this aspect invalidates any semi-classical treatment of the back reaction on the mirror trajectory. Indeed the characteristic mean frequency of the radiation is precisely given in terms of this trajectory. Therefore we are not in presence of "fast" and "slow" variables. But such a division is implicitly assumed in the semi-classical treatment[4] and to my understanding needed for guaranteeing its validity.

We first review the basic properties of the solutions of the Klein Gordon equation in a constant electric field. We work in the homogeneous gauge: $A_t = 0, A_z = Et$. In that case, the momentum p is conserved and the energy $\Omega(p,t)$ of the relativistic mirror of mass M is given by

$$\Omega^2(p,t) = M^2 + (p - Et)^2 \tag{78}$$

Contrariwise to the inertial case, for the accelerating mirror, the fully relativistic equation should be used. Indeed, it is only upon studying the "direct" scattering, i.e. the α coefficient, that one can choose the initial frequency high enough to build a sufficiently localized wave packets with respect to $1/a$. As just said, upon studying pair creation, the frequencies of the produced quanta are given in terms of the classical properties of the mirror's trajectory. Therefore their mean frequency is precisely of the order of the acceleration. This implies that the minimal interval of proper time τ, during which the

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[4]In ref. [17], such a semi-classical treatment has been analyzed. It suffers from several difficulties, such that an acausal behavior induced by a quantum force containing a temporal third derivative of the mirror's trajectory. In ref. [40], it was shown how this acausal behavior disappears when the mirror is taken to be sufficiently transparent at high frequency. In our case, the respect of the Feyman rules upon reflection leads to the decoherence of the emissions. This decoherence can be viewed as a smoothing out of the flux obtained in the absence of recoil. Therefore it eliminates the possibility of defining a local effective force containing a third derivative.

acceleration is constant and which is needed to have a signal not marred by transcients effects, should last many $1/a$. (The same criterion is also required if one wishes to obtain a clean Unruh effect[20], see [13].) The classical relativistic equations of motion of the mirror accelerating to the left are

$$\begin{aligned}
\Omega(p,t) &= M\cosh a\tau \\
t - p/E &= (1/a)\sinh a\tau \\
z - z_0 &= -(1/a)\cosh a\tau
\end{aligned} \tag{79}$$

The free, $\tilde{g} = 0$, Klein Gordon equation for a mode $\psi_p(t,z) = e^{ipz}\chi_p(t)$ is

$$\left[\partial_t^2 + M^2 + (p - Et)^2\right]\chi_p(t) = 0 \tag{80}$$

The solutions are well known, there are cylindrical parabolic (Whittaker) functions, see e.g. [41]. In what follows, we shall suppose that $M^2 >> E$. Then the Schwinger pair production amplitude[42] may be completely ignored since the mean density of produced pairs scales like $e^{-\pi M^2/E}$. Furthermore, the W.K.B. approximation for the modes $\chi_p(t)$ is valid for all t and wave packets of the form

$$\Psi_p(t,z) = \int dp' \frac{e^{-(p'-p)^2/2\sigma^2}}{(\pi\sigma^2)^{1/4}} \psi_{p'}(t,z) \tag{81}$$

do not spread if $\sigma^2 \simeq E$ [41][29]. In that case, the spread in z at fixed t is of the order of $E^{-1/2} = (Ma)^{-1/2}$ for all times and thus much smaller than the acceleration length $1/a$. Therefore these wave packets provide, at least in the absence of scattering, a quantized version of a massive object following a single uniformly accelerated trajectory.

Before analyzing the modifications of these wave packets induced by the spontaneous production of pairs, we compute the matrix elements $\tilde{S}(p, k_\omega, k_\lambda)$. As in eq. (60), to first order in $\tilde{g}$, the amplitude to scatter an initial quantum of momentum k_ω into an outgoing one of momentum k_λ is given by

$$\begin{aligned}
\tilde{S}(p, k_\omega, k_\lambda) &= \delta(p'-p)\delta(k_\lambda - k_\omega) - i\tilde{g}\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dz \langle p'|\psi^\dagger i\overleftrightarrow{\mathcal{D}}_\mu \psi|p\rangle \langle k_\lambda|\phi^\dagger i\overleftrightarrow{\partial}_\mu \phi|k_\omega\rangle \\
&= \delta(p'-p)\delta(k_\lambda - k_\omega) - i\tilde{g}\delta(p'-p-k_\omega + k_\lambda)A(p, k_\omega, k_\lambda)
\end{aligned} \tag{82}$$

We still have momentum conservation owing to the homogeneous character of the external electric field. The amplitude $A(p, k_\omega, k_\lambda)$ is given by

$$A(p, k_\omega, k_\lambda) = \int_{-\infty}^{\infty} dt \left[\chi_{p+k_\omega-k_\lambda}^*(t) i\overleftrightarrow{\mathcal{D}}_\mu \chi_p(t)\right](k_\omega^\mu + k_\lambda^\mu)\frac{e^{i(\lambda-\omega)t}}{4\pi\sqrt{\omega\lambda}} \tag{83}$$

Using the W.K.B. solutions of eq. (80), $A(p, k_\omega, k_\lambda)$ becomes

$$A(p, k_\omega, k_\lambda) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i\int^t dt'[\Omega(p+k_\omega-k_\lambda,t')-\Omega(p,t')]}e^{i(\lambda-\omega)t}$$

$$\left[\frac{(\Omega(p+k_\omega-k_\lambda,t) + \Omega(p,t))(\omega+\lambda) - (2p+k_\omega-k_\lambda+2Et)(k_\omega+k_\lambda)}{4\sqrt{\Omega(p+k_\omega-k_\lambda,t)\Omega(p,t)\omega\lambda}}\right] \tag{84}$$

compare with the inertial case, eq. (61). Using the classical equations (79), developing the phase to first order in $k_\omega - k_\lambda$ and neglecting the dependance in $k_\omega - k_\lambda$ both in the numerator and in the denominator when it appears added to Et, one obtains

$$A(p, k_\omega, k_\lambda) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-i(k_\omega - k_\lambda)z_{cl}(t) + i(\lambda - \omega)t} \left[\frac{E z_{cl}(t)(\omega + \lambda) - (Et - p)(k_\omega + k_\lambda)}{2\Omega(p, t)\sqrt{\omega\lambda}} \right] \tag{85}$$

See ref. [16] for a detailed analysis of the semi-classical properties of the W.K.B. waves. To this order in $k_\omega - k_\lambda$, we emphasize that only classical physics enters into the amplitude $A(p, k_\omega, k_\lambda)$. Indeed, if ω is a V mode and λ a U mode, i.e $k_\omega = -\omega, k_\lambda = \lambda$, using $U = t - z_{cl}(t)$ as the dummy variable to perform the integration, one recovers identically (up to an irrelevant phase) the $\alpha_{\omega,\lambda}$ coefficient of the no recoil case, eq. (33). Therefore the quantum properties are encoded in higher orders in the momentum transfer.

Since the mirror is subject to the electric field, there is now spontaneous pair creation. The amplitude of probability to create a pair a quanta with momenta k_ω and k_λ is

$$\begin{aligned}
\tilde{S}_{creat}(p, k_\omega, k_\lambda) &= -i\tilde{g} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dz \langle p' | \psi^\dagger i \overleftrightarrow{\mathcal{D}}_\mu \psi | p \rangle \langle 0 | a_{k_\lambda} b_{k_\omega} \phi^\dagger i \overleftrightarrow{\partial}_\mu \phi | 0 \rangle \\
&= -i\tilde{g}\delta(p' - p + k_\omega + k_\lambda) B(p, k_\omega, k_\lambda) \tag{86}
\end{aligned}$$

where the amplitude $B(p, k_\omega, k_\lambda)$ is given by (compare with eq. (83))

$$\begin{aligned}
B(p, k_\omega, k_\lambda) &= \int_{-\infty}^{\infty} dt \left[\chi^*_{p - k_\omega - k_\lambda}(t) i \overleftrightarrow{\mathcal{D}}_\mu \chi_p(t) \right] (k_\lambda^\mu - k_\omega^\mu) \frac{e^{i(\lambda + \omega)t}}{4\pi\sqrt{\omega\lambda}} \\
&= A(p, -k_\omega, k_\lambda) \\
&= \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(k_\omega + k_\lambda)z_{cl}(t) + i(\lambda + \omega)t} \left[\frac{E z_{cl}(t)(-\omega + \lambda) - (Et - p)(-k_\omega + k_\lambda)}{2\Omega(p, t)\sqrt{\omega\lambda}} \right] \tag{87}
\end{aligned}$$

where we have performed the same approximations which have lead to eq. (85). Using again the variable $U = t - z_{cl}(t)$, we recover $\beta^*_{\omega,\lambda}$ of eq. (33). We insist on the fact that $\tilde{S}_{creat}(p, k_\omega, k_\lambda)$ is interpretated as the amplitude of probability to create the pair of quanta b_ω and a_λ. On the contrary, $\beta_{\omega,\lambda}$ had not such a simple interpretation: in terms of matrix elements of in and out operators, it is given by

$$\beta_{\omega,\lambda} = \langle 0, in | a_\lambda^{\dagger out} b_\omega^{\dagger in} | 0, in \rangle \tag{88}$$

It is hard to interpret the matrix element as a given pair creation act since the frequency ω, the in operator and the in-vacuum are defined in the remote past. Furthermore, if one wishes to obtain this matrix element from the dynamical model, it would require presumably a resummation of a certain class of Feyman diagrams each of which being evaluated approximatively (no loop). To determine the class of diagrams and the approximations used seems a difficult task but will presumably shed light onto the approximate nature of the semi-classical treatment.

We have thus shown that to first order in $\tilde{g}$ and in the momentum transfer $k_\omega \pm k_\lambda$, the matrix elements $A(p, k_\omega, k_\lambda)$ and $B(p, k_\omega, k_\lambda)$ of the dynamical model coincide with the Bogoliubov coefficients $\alpha_{\omega,\lambda}$ and $\beta_{\omega,\lambda}$ obtained in the DF model.

We now consider higher orders in the momentum transfer. We evaluate these higher orders using the saddle point approximation. The stationary phase condition of the amplitude $A(p, k_\omega, k_\lambda)$ indicates at which time t^* one has conservation of the Minkowski energy:

$$\omega + \Omega(p, t^*) = \lambda + \Omega(p + k_\omega - k_\lambda, t^*) \tag{89}$$

Taking the square of eq. (89) and using eqs. (78, 79), one obtains

$$
\begin{aligned}
\omega\left[\Omega(p, t^*) + p - Et^*\right] &= \lambda(\Omega(p, t^*) - p + Et^* + 2\omega) \\
\omega e^{-a\tau^*} &= \lambda(e^{a\tau^*} + 2\omega/M)
\end{aligned}
\tag{90}
$$

This resonance condition between ω and λ differs from the no recoil result, given by eq. (13) with $V_{cl} = -1/a^2 U$, by the additional term on the r.h.s.: $2\lambda\omega/M$. This term is quantum in character since it is linear in $\hbar$. Its origin can be attributed to the "absorption" of the ω quantum which precedes the emission of the λ quantum. Indeed if one study, as in the accelerated detector case, the double process (described by a cubic interaction acting twice) of an absorption followed by an emission, one finds eq. (89) upon eliminating the energy of the intermediate excited state[16].

Then straightforward algebra gives the saddle point approximation of A and B. For $\omega >> \lambda$, i.e. in the future branch of the hyperbola, eq. (79), one obtains

$$
\begin{aligned}
A^{s.p.}(p, \omega, \lambda) &= \alpha^{s.p.}_{\omega, \lambda} \, e^{i[-2\omega p + \omega^2]/2E} \\
B^{s.p.}(p, \omega, \lambda) &= \beta^{s.p.*}_{\omega, \lambda} \, e^{i[2\omega p + \omega^2]/2E}
\end{aligned}
\tag{91}
$$

where $\alpha^{s.p.}_{\omega, \lambda}, \beta^{s.p.}_{\omega, \lambda}$ are the saddle point expressions for the integrals given in eqs. (33). The additional phases are quantum in character. It is now appropriate to make contact with ref. [43] in which the quantum corrections to the Bogoliubov coefficients evaluated at the background field approximation are also considered. We might then say that the factors appearing in eqs. (91) arise from the "quantization of the Bogoliubov coefficients" here represented in their quantized version by the amplitudes A and B. These relations between saddle point expressions can be extended to relations between the exact expression of A (B) with the exact one of α (β) given in eq. (33). This is done in the Appendix. The exact relations confirm the validity of the saddle point treatment. What interests us is to determine the consequences of these additional phases upon computing the mean flux and the correlations among emitted quanta.

We first study the mean flux. We wish to compare, to order g^2, the flux emitted by the dynamical mirror with the flux obtained in the no recoil case given in eqs. (22, 25). In that case, for the uniformly accelerated mirror, we recall that the mean flux vanishes identically and that the first term of eq. (25) is constant in the rest frame of the mirror. To compute the flux in the dynamical case, it suffices to transpose what was done in Section 4 where we studied the scattering by an inertial mirror to the present case of spontaneous production.

In that section, we saw that a too small σ, the inverse spread of the mirror, easily leads to decoherence since this effect appears once the scattered frequency ω is bigger than σ. We also mentioned that for too large σ, one should take into account the energy

of the mirror, see discussion after eq. (73). Both restrictions appear in the present accelerated case and play a determinative role. Indeed, their combined effects are such that the decoherence of successive emissions necessarily appears before a proper time lapse given by $\ln(M/a)/2a$ no matter what spread σ has been chosen. More precisely, the period of coherence is maintained during that interval only if one deals with minimal wave packets characterized by $\sigma^2 = E$. In order to prove this, we use the wave packets given in eq. (81). We center the mean initial momentum at $p = 0$ and choose the phase of the W.K.B solutions in such a way that the position of the turning point of the mean trajectory encoded in the initial wave function is at $t = 0, z_0 = 0$, see eq. (79). This choice of phases amounts to set the interaction at $t = 0$ and to look only into the future left moving part of the hyperbola eq. (79). Therefore the frequency ω of the V mode is blue-shifted while the frequency λ of the U mode in red shifted. Then only V quanta will induce decoherence effects.

We can anticipate these effects. When a minimal wave packet is scattered by the production of a pair in which the V quantum has an energy greater than $\sigma = E^{1/2}$, we find, as in eq. (71), that the scattered wave packet scattered no longer overlaps with the unscattered one. Furthermore, in the rest frame of the mirror, the mean frequencies of the quanta spontaneously produced are of the order of a. Therefore, after a proper time greather $2a\tau = \ln(M/a)$, the frequency of the quanta emitted forward are greather than $E^{1/2}$. Then the successive emissions of forward quanta are completely incoherent and a positive flux is emitted. On the contrary, $\langle T_{UU} \rangle$, the flux emitted backward, is still coherent and thus still vanishes. This will be proven explicitly to order $\tilde{g}^2$; we shall see how the recoil induced by the blue shifted V-partner disappears from the expression of $\langle T_{UU} \rangle$.

The proof of these facts goes along the same lines as the development from eqs. (69 $\rightarrow$ 73). The initial state is now $|in'\rangle = |Mir\rangle|0\rangle$, see eq. (64) and eq. (81). The scattered state has the following structure

$$\tilde{S}|in'\rangle = |in'\rangle - i\tilde{g}|Mir', 1pair\rangle - \tilde{g}^2|Mir'', 1scattered\ pair\rangle \tag{92}$$

where the linear term in $\tilde{g}$ is given by

$$|Mir', 1pair\rangle = \int dk_\omega \int dk_\lambda \int dp\ h_p\ B(p, k_\omega, k_\lambda)c^\dagger_{p-k_\omega-k_\lambda}|0\rangle_{Mir} \otimes a^\dagger_{k_\lambda}b^\dagger_{k_\omega}|0\rangle \tag{93}$$

see eq. (88) for the amplitude $B(p, k_\omega, k_\lambda)$ to create the ω, λ pair. We have extracted from the terms in $\tilde{g}^2$ the part that shall contribute to the mean flux to order $\tilde{g}^2$. This part contains only one created pair which has been scattered after having been created. Neglecting once more the complication of the time ordered product of $\tilde{H}_{int}$, one finds

$$|Mir'', 1scattered\ pair\rangle = \int dk_\omega \int dk_\lambda \int d\lambda' \int dp\ h_p\ B(p, k_\omega, k_\lambda)A(p - k_\omega - k_\lambda, k_\omega, k'_\lambda)$$
$$c^\dagger_{p-k'_\lambda-k_\lambda}|0\rangle_{Mir} \otimes a^\dagger_{k_\lambda}b^\dagger_{k'_\lambda}|0\rangle + (\lambda \rightarrow \omega) \tag{94}$$

where the amplitude $A(p - k_\omega - k_\lambda, k_\omega, k'_\lambda)$ is given in eq. (85). This second scattering has replaced $b^\dagger_{k_\omega}$ by a $b^\dagger_{k'_\lambda}$ in the first term and $a^\dagger_{k_\lambda}$ by $a^\dagger_{k'_\lambda}$ in the second one.

Then, to order $\tilde{g}^2$, the mean energy emitted to the left is

$$\langle \tilde{H}_V \rangle = \langle in'|\tilde{S}^\dagger H_V \tilde{S}|in'\rangle = \tilde{g}^2 \int dp |h_p|^2 \int_0^\infty d\omega\omega \int_0^\infty d\lambda |B(p,\omega,\lambda)|^2$$

$$= \tilde{g}^2 \int_0^\infty d\omega\omega \int_0^\infty d\lambda |\beta(\omega,\lambda)|^2 \tag{95}$$

As in eq. (70), only the linear term in $\tilde{g}$ contributes to $\langle \tilde{H}_V \rangle$. Then the fact that $B(p,\omega,\lambda)$ differs from $\beta(\omega,\lambda)$ by a phase only, see eq. (91), guarantees that the mirror's wave function factorizes out and that the total energy is unaffected by the recoils.

On the contrary the local flux emitted to the left is affected by these phases. It is given by

$$\langle \tilde{T}_{VV} \rangle = \langle in'|\tilde{S}^\dagger T_{VV}\tilde{S}|in'\rangle = \langle \tilde{T}_{VV} \rangle_1 + \langle \tilde{T}_{VV} \rangle_2 \tag{96}$$

$\langle \tilde{T}_{VV} \rangle_1$ is quadratic in B

$$\langle \tilde{T}_{VV} \rangle_1 = 2\tilde{g}^2 \int dp_1 \int dp_2 \frac{e^{-(p_1)^2/2\sigma^2}}{(\pi\sigma^2)^{1/4}} \frac{e^{-(p_2)^2/2\sigma^2}}{(\pi\sigma^2)^{1/4}} \int_0^\infty d\omega \int_0^\infty d\omega' \int_0^\infty d\lambda \int_0^\infty d\lambda' \delta(\lambda-\lambda')$$

$$\delta(p_1+\omega-p_2-\omega') \left[B(p_1,\omega,\lambda)B^*(p_2,\omega',\lambda')\frac{\sqrt{\omega\omega'}}{2\pi}e^{i(\omega'-\omega)V} \right] \tag{97}$$

where the δ's of Dirac comes from momentum conservation of the mirror and the U quantum of frequency λ. Similarly the interfering term, $\langle \tilde{T}_{VV} \rangle_2$ is

$$\langle \tilde{T}_{VV} \rangle_2 = 2\tilde{g}^2 \int dp_1 \int dp_2 \frac{e^{-(p_1)^2/2\sigma^2}}{(\pi\sigma^2)^{1/4}} \frac{e^{-(p_2)^2/2\sigma^2}}{(\pi\sigma^2)^{1/4}} \int_0^\infty d\omega \int_0^\infty d\omega' \int_0^\infty d\lambda \int_0^\infty d\lambda' \delta(\lambda-\lambda')$$

$$\delta(p_1+\omega+\omega'-p_2)\mathrm{Re} \left[B(p_1,\omega,\lambda)A(p_1+\omega-\lambda,\omega',\lambda')\frac{-\sqrt{\omega\omega'}}{2\pi}e^{-i(\omega'+\omega)V} \right] \tag{98}$$

where $B(p_1,\omega,\lambda)A(p_1-\omega+\lambda,\omega',\lambda')$ is the amplitude to emit two V photons of frequencies ω and ω', see eq. (94). Notice that the argument of the δ of Dirac arising from the overlap of the mirror states is not the same as in eq. (97). It comes now from the overlap between the twice scattered mirror of momentum $p_1+\omega+\omega'$ with the unperturbed one of momentum p_2.

Using eqs. (91), one can perform the integrals over p_2 and p_1. One obtains

$$\langle \tilde{T}_{VV} \rangle_1 = 2\int_0^\infty d\omega \int_0^\infty d\omega' e^{-(\omega-\omega')^2(1/\sigma^2+\sigma^2/E^2)/4} \int_0^\infty d\lambda \left[\beta_{\omega,\lambda}^{s.p.*}\beta_{\omega',\lambda}^{s.p.}\frac{\sqrt{\omega\omega'}}{2\pi}e^{i(\omega'-\omega)V} \right]$$

$$= \int_{-\infty}^\infty d\eta \frac{e^{-\eta^2/(1/\sigma^2+\sigma^2/E^2)}}{(\pi(1/\sigma^2+\sigma^2/E^2))^{1/2}} \langle T_{VV}(V+\eta)\rangle^{first\ term} \tag{99}$$

and

$$\langle \tilde{T}_{VV} \rangle_2 = -2\int_0^\infty d\omega \int_0^\infty d\omega' e^{-(\omega+\omega')^2(1/\sigma^2+\sigma^2/E^2)/4} \int_0^\infty d\lambda \mathrm{Re} \left[\beta_{\omega,\lambda}^{s.p.*}\alpha_{\omega',\lambda}^{s.p.}\frac{\sqrt{\omega\omega'}}{2\pi}e^{-i(\omega'+\omega)V} \right]$$

$$= \int_{-\infty}^\infty d\eta \frac{e^{-\eta^2/(1/\sigma^2+\sigma^2/E^2)}}{(\pi(1/\sigma^2+\sigma^2/E^2))^{1/2}} \langle T_{VV}(V+\eta)\rangle^{second\ term} \tag{100}$$

In the first equalities, the quantities in brackets are computed in the no-recoil case in the saddle point approximation, see eq. (33). In the second equalities, $\langle T_{VV} \rangle^{first\ term}$ and $\langle T_{VV} \rangle^{second\ term}$ are respectively the first and second term of eq. (25). Notice that the width of the gaussian is minimal for minimal wave packets, i.e. $\sigma^2 = E$, and that the dependence in E came from the quantum phases of eqs. (91) only.

Thus to order $\tilde{g}^2$, by neglecting the time ordering upon evaluating the scattered state in $\tilde{g}^2$, and by assuming that $M >> a$, the mean flux emitted by the dynamical mirror can be expressed as the integral of the flux of the no-recoil case over a region given by $(1/\sigma^2 + \sigma^2/E^2)^{1/2}$ where $1/\sigma$ is the spread of the initial wave packet. Most presumably no simple relation will be found when one of the conditions listed above is not fulfilled. Nevertheless, the second interfering term will always be erased by the recoils since nothing can prevent momentum transfer nor affect the exponentially growing Doppler shift relation between in and out frequencies. Therefore, once $\omega > E^{1/2}$, or $2a\tau^* > \ln M/a$, see eq. (90), $\langle \tilde{T}_{VV} \rangle_2$ vanishes and the flux emitted forward is now positive[5] and incoherent since the gaussian factor $e^{-(\omega - \omega')^2 (1/\sigma^2 + \sigma^2/E^2)/4}$ appearing in $\langle \tilde{T}_{VV} \rangle_1$ restricts ω' to be equal to $\omega \pm (1/\sigma^2 + \sigma^2/E^2)^{-1/2}$.

Similar equations give $\langle \tilde{T}_{UU} \rangle$, the mean flux emitted to the right by Doppler red shifted U quanta. It is important to notice that despite the presence of hard blue shifted V quanta, the fact that T_{UU} acts as the operator unity for the V quanta leads to a $\delta(\omega - \omega')$ in the place of the $\delta(\lambda - \lambda')$ of eq. (98). This delta insures that the quantum phases of the amplitudes B and A either cancel out or are reduced to low and negligible corrections when $M >> a$. Thus $\langle \tilde{T}_{UU} \rangle$ vanishes as in the DF model. It would be interesting to generalize this decoupling to all order in $\tilde{g}$ and to understand how generic is this decoupling. This is because a similar decoupling might arise in the black hole context as well. Indeed in that case, the out going frequencies are exponentially blue shifted[1] as one approaches the Schwarzschild horizon at fixed advanced Eddington Finkelstein v and are gravitationally coupled to the soft infalling modes. Up to now there is no clear understanding of whether these interactions are negligible[44], thereby legitimizing the semi classical approximation, or big[3][11][43][12][14] and invalidating the semi classical predictions.

To further investigate the consequencies of the introduction of the mirror's dynamics, we now consider the correlations between some specific emitted quantum and the state of the mirror.

We start by isolating the "partner" wave function of a specific quantun. As in eq. (28), we use a projector on the specific quantum. Since the mirror is dynamical, the projector is now enlarged to

$$\Pi_\lambda^{dyn} = \ \mathrm{I}_{Mir} \otimes \ \mathrm{I}_b \otimes \int_0^\infty d\lambda \ f_\lambda a_\lambda^\dagger |0_a\rangle\langle 0_a| \int_0^\infty d\lambda' \ f_{\lambda'}^* a_{\lambda'} \tag{101}$$

---

[5]The same conclusion is obtained when one studies the electro-magnetic flux radiated by a charged particle in a constant electric field, see [38][39]. This is not in contradiction with the vanishing classical reaction force in the case of uniform acceleration. In the second case, the trajectory is *a priori* given and thus it is not a solution of the coupled Euler Lagrange equations for the particle and the A_μ field. The important point is that one cannot use the vanishing result of the second case to claim that an electron in a constant E-field does not radiate, even locally.

where I_{Mir} is the operator unity on the mirror's Fock space. To order $\tilde{g}^2$, the probability to find such a state is given by $\tilde{g}^2 P_{\Pi_\lambda}$, see eq. (29), by virtue of eqs. (91) which relate the dynamical matrix elements to the former Bogoliubov coefficients. Again one sees that the fact that the probability is a sum of squares leads to almost no modification even when λ describes a V quantum whose energy is much bigger than M. This is because the corrections of the norm of the amplitudes are given by terms in $\lambda^n \partial_\lambda^n \ln \Omega(p - \lambda, t)$. These remain of the order of a/M for all times.

Nevertheless, the "partner" of the quantum $|\bar{\lambda}\rangle = \int d\lambda f_\lambda a_\lambda^\dagger |0_a\rangle$ is no longer given by eq. (30) but by

$$
\begin{aligned}
|partner, \ \bar{\lambda}\rangle^{dyn} &= \langle 0_a| \int d\lambda f_\lambda^* a_\lambda \ \tilde{S}|0\rangle|Mir\rangle \\
&= \int_{-\infty}^{\infty} dp h_p \int_0^{\infty} d\lambda f_\lambda^* \int_0^{\infty} d\omega B(p, \omega, \lambda) c_{p+\omega-\lambda}^\dagger |0\rangle_{Mir} \otimes b_\omega^\dagger |0_b\rangle (102)
\end{aligned}
$$

see eq. (93). We see that we must have both ω and λ are much smaller than σ, the width in p encoded in h_p, in order to be allowed to factorize out the initial mirror's wave function $|Mir\rangle$, eq. (64), without enjuring the result. In that case, one recovers exactly the partner as defined in eq. (30). On the contrary, when $\omega - \lambda$ is comparable to σ, we can no longer factorize out the mirror wave function and the "partner" is truly a combined (entangled) state of the anti particle and the mirror's state. This is precisely what leads to decoherence and what happened for the uniformly accelerated mirror since the forward frequencies are blue shifted according to $\omega = ae^{a\tau^*}$, see eq. (90).

Another way to investigate the correlations is to consider the conditional value of $T_{\mu\nu}$ correlated to a particular final state. As stressed in [13], the matrix elements $\langle T_{\mu\nu}\rangle_\Pi$ given in eq. (31) change in time from an off-diagonal matrix element when one inquires into the value of $T_{\mu\nu}$ before the interaction occurs to a more usual expectation value when one evaluates the conditional value after the interaction has occurred. Therefore, by virtue of the fact that the additional phases introduced by the recoils play an important role only in interfering situations, the conditional value of $T_{\mu\nu}$ is completely washed out in the past once the frequency ω or λ is greater than σ. On the contrary, in the future, the conditional value stays almost unaffected by the recoil of the mirror. The easiest way to understand this double behavior, is to compare the momentum transfer to the mirror in both situations.

We consider the case in which one selects a U quantum by the projector Π_λ^{dyn}, see eq. (101), and we evaluate the conditional value of T_{VV} to order $\tilde{g}^2$, see eq. (74). We first analyze the structure of $\langle T_{VV}\rangle_\Pi$ before the interaction occurs, i.e. on the right of the mirror. We obtain that the interacting hamiltonian acts linearly on both sides of the projector Π_λ^{dyn}. On its right, one finds a matrix element of this type: $\langle Mir'|\langle\bar{\lambda}|\langle\omega|\tilde{S} \ T_{VV}|0\rangle|Mir\rangle$ since one starts from $|0\rangle|Mir\rangle$ and select the state $|\bar{\lambda}\rangle$. The state $\langle\omega|$ represent the unscattered V quantum created by T_{VV}. Therefore the momentum of the mirror in the bra $\langle Mir'|$ is $p - \omega' - \bar{\lambda}$ where ω' is the other V quantum created by T_{VV} acting on Minkowski vacuum. This latter gets reflected with amplitude $A(p, \omega', \bar{\lambda})$, eq. (82), to constitute the selected U quantum $|\bar{\lambda}\rangle$. On the left of the projector Π_λ^{dyn}, the matrix element is of the form $\langle Mir|\langle 0|\tilde{S}|\bar{\lambda}\rangle|\omega\rangle|Mir''\rangle = B(p, \omega, \bar{\lambda})$. In the ket $|Mir''\rangle$, the momentum of the

mirror is $p + \omega - \bar{\lambda}$ since both ω and $\bar{\lambda}$ are created. Therefore the overlap between the two states of the mirror vanishes once the V quantum $\omega + \omega'$ is greather than σ exactly as for the $\langle T_{VV} \rangle_2$ term in eqs. (73, 100).

We now consider $\langle T_{VV} \rangle_\Pi$ after the creation act, on the left of the mirror. As before we find that the interacting hamiltonian acts linearly on both sides of the projector. But the matrix element on the right of the projector has now the following structure: $\langle Mir' | \langle \omega | \langle \bar{\lambda} | T_{VV} \tilde{S} | 0 \rangle | Mir \rangle$. This matrix element is governed by the amplitude $B(p, \omega', \bar{\lambda})$ to create the pair $\bar{\lambda}, \omega'$ and the operator T_{VV} replaces the quantum ω' by ω. On the left hand side of the projector nothing changes. Thus the mirror's momentum is now $p + \omega' - \bar{\lambda}$ in the bra $\langle Mir' |$ and still $p + \omega - \bar{\lambda}$ in the ket $| Mir \rangle$. Then the overlap between the two states of the mirror constraints $\omega - \omega'$. This is exactly what happened for $\langle T_{VV} \rangle_1$ in eq. (99).

Thus we have proven that the conditional value behaves, in the past, like an interfering "washable" term in which the sum of the blue shifted frequencies appear, and, in the future, like the first term in the decomposition of the mean in which the difference of the momenta occur. It is also important to realize that the situation for the conditional value of T_{UU} is not the same even though one obtains expressions with exactly the same structure under the replacement of the V momentum ω by the U momentum λ. For $\langle T_{UU} \rangle_\Pi$, the momentum transfer that made $\langle T_{VV} \rangle_\Pi$ to vanish in the past, i.e. 2ω, is now replaced by 2λ which is a very low Doppler red shifted frequency. Therefore, the conditional value of T_{UU} is unchanged (recall that one considers only the future part of the hyperbola, for $t > t_{t.p.}$, see eq. (79)). For the same reasons, we found that the mean flux in the U direction was unchanged, see discussion in the paragraph before eq. (101).

Conclusions

In this article we showed how decoherence effects occur when one takes into account the dynamics of the scattered agent and how they modify the properties of the matrix elements of $T_{\mu\nu}$. In particular, when the frequency of the spontaneously produced quanta grows due to some Doppler effect, these effects inevitably lead to an incoherent positive flux as well as to vanishing conditional values of the flux in the past of the creation act. We believe that both consequences are generic in the sense that they will also occur in cosmological contexts and in the black hole evaporation process. Indeed in both cases, in the background field approximation, when one traces backwards a quantum, its frequency is blue shifted according to the gravitational properties of the background. This classical relation will certainly be modified once the momentum energy of the quantum will dominate the background mean energy. Then the local correlations between the presence of this late quantum and the field configurations at early times which are found when one uses the background field approximation will be equally washed out by the recoil of the geometry treated dynamically.

The crucial point which remains to be done is to determine the consequences of the modifications induced by these recoil effects. Indeed, to the lowest order in $\tilde{g}$, we saw that the probabilities to find a specific event as well as the expectation values expressible as sum of squares of amplitutes are almost unaffected by the recoil effects. Therefore,

one must determine if the higher orders in $\tilde{g}$ will sufficiently modify the dynamics at later times that these expectation values will no longer be correctly approximated by their lowest order expressions.

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6 Appendix: The Exact Amplitudes

The goal of this Appendix is to confirm the validity of eqs. (91) which relate the saddle point expressions for the scattering amplitudes A (B) to the saddle point expressions for the Bogoliubov coefficients α (β) by finding a similar relations between exact expressions for A and for α.

To this end we shall use the integral representations of the exact solutions of eq. (80) rather than the W.K.B. approximation which were used in eq. (85). The integral representation of an in mode with asymptotic unit initial current coming from $z = -\infty$ is given by

$$\chi_p^{in}(t) = N \int_{-\infty}^{0} du(-u)^{\frac{iM^2}{2E}-\frac{1}{2}} e^{-iE\left[u^2/4+(t-p/E)u+(t-p/E)^2/2\right]} \tag{103}$$

where N^{-1} is given by $e^{\pi M^2/E}\Gamma(1/2 + iM^2/2E)$, see [45]. The dummy variable u is classically related to the time t and the energy $\Omega(p,t)$, eq. (78), by $u = t - Ep - \Omega(P,t)$. $u/\sqrt{2E}$ plays for this inverse harmonic potential case a role very similar to the annihilation operator $a = (p + iq)/\sqrt{2}$ of the harmonic oscillator. Similarly, the integral representation of an out mode with asymptotic unit final current directed towards $z = -\infty$ is

$$\chi_p^{out}(t) = N \int_{-\infty}^{0} du(-u)^{-\frac{iM^2}{2E}-\frac{1}{2}} e^{iE\left[u^2/4-(t-p/E)u+(t-p/E)^2/2\right]} \tag{104}$$

see [45] for more details.

Therefore the amplitude $A(p,\omega,\lambda)$, see eq. (83), is given by

$$
A(p,\omega,\lambda) = N^2 \int_{-\infty}^{\infty} \frac{d\tilde{t}}{4\pi} \int_{-\infty}^{0} du_1 \int_{-\infty}^{0} du_2 \frac{e^{-i(\omega-\lambda)\tilde{t}}}{\sqrt{\omega\lambda}} e^{-i(\omega-\lambda)p/E} (-u_1)^{\frac{iM^2}{2E}-\frac{1}{2}} (-u_2)^{\frac{iM^2}{2E}-\frac{1}{2}}
$$
$$
(8\lambda i\partial_{u_1} e^{-iE\left[u_1^2/4+\tilde{t}u_1+\tilde{t}^2/2\right]}) e^{-iE\left[u_2^2/4-(\tilde{t}-\omega/E-\lambda/E)u_2+(\tilde{t}-\omega/E-\lambda/E)^2/2\right]} \tag{105}
$$

where we have defined $\tilde{t} = t - p/E$. The derivative factor $8\lambda i\partial_u$ is a useful reexpression of the matrix elements of the current operators $J_\phi^\mu J_\mu^{A,\psi}$. Performing the gaussian integration over $\tilde{t}$, one gets

$$
A(p,\omega,\lambda) = \frac{N^2}{\sqrt{2\pi E}} \frac{e^{-i(\omega-\lambda)p/E}}{\sqrt{\omega\lambda}} \int_{-\infty}^{0} du_1 \int_{-\infty}^{0} du_2 (-u_1)^{\frac{iM^2}{2E}-\frac{1}{2}} (-u_2)^{\frac{iM^2}{2E}-\frac{1}{2}}
$$
$$
(4\lambda i\partial_{u_1} e^{-iE\left[u_1 u_2/2+u_1\omega/E+u_2\lambda/E-\omega^2/E^2+(\omega-\lambda)^2/2E^2\right]}) \tag{106}
$$

Introducing the variable $\delta = (E/M)^2 u_1 u_2 = a^2 u_1 u_2$ and integrating par part, one has

$$
\begin{aligned}
A(p,\omega,\lambda) &= e^{-i(\omega-\lambda)p/E} e^{i\omega^2/E - i(\omega-\lambda)^2/2E} \frac{N^2}{\sqrt{2\pi E}} (M - ia) \int_0^\infty d\delta \; \delta^{\frac{iM^2}{2E} - \frac{3}{2}} e^{-\frac{iM^2\delta}{2E}} \\
&\qquad \int_{-\infty}^0 du_2 \; e^{-i\lambda u_2} e^{-i\omega\delta/a^2 u_2} \frac{2\lambda}{\sqrt{\omega\lambda}} \\
&= e^{-i(\omega-\lambda)p/E} e^{i\omega^2/E - i(\omega-\lambda)^2/2E} \frac{2\pi N^2}{\sqrt{2\pi E}} (M - ia) \int_0^\infty d\delta \; \delta^{\frac{iM^2}{2E} - 1} e^{-\frac{iM^2\delta}{2E}} \\
&\qquad (\frac{1}{a\pi}) K_1 \left[2i(\frac{\omega\lambda\delta}{a^2})^{1/2} \right]
\end{aligned}
\tag{107}
$$

see eq. (33) for the definition of the Bessel function. In the limit $M^2/E \to \infty$ at fixed $a = E/M$ (i.e. $\hbar \to 0$) and for $\omega\lambda/a^2 = \mathcal{O}(1)$, the integral over δ gets its contribution from the saddle region $\delta = 1 \pm (a/M)^{1/2}$ and the normalisation factor $N \to 1/\sqrt{2\pi}$. In that case, one finds

$$
A(p,\omega,\lambda) = e^{-i(\omega-\lambda)p/E} e^{i(\omega^2-\lambda^2)/2E} \alpha(\omega,\lambda)
\tag{108}
$$

Therefore, when $\omega >> \lambda$, one recovers eq. (91). It is interesting to notice that the dummy variable u_2 is classically equal to mirror's coordinate $U = -e^{a\tau}/a$ and plays exactly the role of this classical variable in eq. (107). On the contrary the variable $\delta = a^2 u_1 u_2$ is classically equal to 1 (since u_1 is classically equal to $-V_{cl}(U) = 1/a^2 U$) and acts as a quantum spread around the classical trajectory. This spread is controlled by $(M/\hbar a)^{-1/2}$ and has nothing to do with the spread in position of the initial wave packet, rather it is here induced by the specification of ω and λ as well as the conservation of momentum and energy (forced by the integral over $\tilde{t}$).

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